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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

Volume XLII Number 3

Edited by William David Reeve

Teaching Computation with Approximate Data*

By C. N. SHUSTER

State Teachers College, Trenton, N. J.

SEEING THE PROBLEM: In the last four years the author has asked 276 students to find the square root of 64.000. These students ranged from high school graduates in the upper half of their classes up to teachers in high schools, teacher training schools, and colleges. Only seven of these superior students, or less than 3% got the correct answer (8.0000). The favorite answers were ± 8, ± 8.0, and ± 8.00.

Last year the author visited the class of a college professor who loves to talk of rigor. Using two digit and three digit data he obtained an answer with six digits. Of course there was no possibility that his last four digits were correct. This unfortunate situation is duplicated daily in dozens of high school and college classes in every state.

Mathematics that produces incorrect answers to every practical problem is worse than useless. Because of this the topic of computation with approximate data is easily the most important topic in high school mathematics. It would be very easy to teach this topic along with a far better treatment of decimal fractions, growing out of decimal measuring, in the sixth, seventh, and eighth grades. However we do not have, in these grades, many teachers who understand the topic sufficiently well to teach it and we do not have the proper text books. In ninth grade algebra we spend many months teaching factoring and its so called applications. At least 85% of this work is a senseless waste of time. If we reduced the time spent on factoring and similar questionable topics we could include a thorough treatment of computation with approximate data, the slide rule, numerical trigonometry and many other vital new topics in the ninth grade.

Computation with approximate data is needed in a large proportion of the problems of life, since these problems contain approximate data secured from measurements or are taken from various handbook tables. The table on the properties of saturated steam in the *Handbook of Physics and Chemistry* (9) contains 3,591 approximate factors, and there are hundreds of other tables in the dozens of similar handbooks. In solving problems containing approximate data, one should always follow the rules for computing with approximate data.

Pupils can measure to the nearest foot, to the nearest 0.1 foot, or to the nearest 0.01 foot, according to the graduations on the tape and the care with which it is used, but they cannot measure any length exactly. There are at least ten sources of

* This paper was requested for the January 1949 issue, but could not be included because of illness of the author—Editor.

error in measuring so simple a thing as the distance between two points several hundred feet apart. Pupils should learn that no one in the whole history of the world has ever made an exactly accurate measurement of any length. The same is true of measurements of time, weight, volume, area, temperature, rotation, latitude, longitude, and the like. The most accurate measurements ever made do no have more than eight significant digits, and most of the measurements used in industry have from three to five significant digits.

Fundamental Concepts. Pupils should be taught the meaning of *unit of measurement* and *significant digits* and *how to round numbers* and *when* and *why* this is necessary. The *unit of measurement is the smallest unit used in a measurement.* The *unit of measurement* in the following measurements is given in the parentheses: 64.32 feet (0.01 foot); 43.8 feet (0.1 foot); 34.3. pounds (0.1 pound); 16 pounds 12 ounces (1 ounce); 34°24′16″ (1″); $16\frac{3}{8}$ inches ($\frac{1}{8}$ inch); 6,843 miles (1 mile); 10.2 seconds (0.1 second); 2.1034 inches (0.0001 inch).

If a distance has been measured so accurately that it is known to be nearer to 986.3 feet than it is either to 986.2 or to 986.4 feet, the measurement has four significant digits. *For all digits of a number to be significant, every digit except the last must be correct, and the error in the last digit must not be greater than one half the unit of measurement.*

If a measurement 26.8 inches has three significant digits, it must be between 26.75 inches and 26.85 inches. If the measurement 165.37 feet has five significant digits, it must be between 165.365 feet and 165.375 feet. Zeros that are the result of correct measurement are significant. If the measurement 300 feet is correct to the nearest foot, or lies between 299.5 feet and 300.5 feet, the measurement has three significant digits and the two zeros are both significant. Zeros should not be written after a decimal point in a mixed decimal unless they are significant. One can be sure that the measurement 26.00 feet has four significant digits and that the measurement 3.0000 inches has five significant digits.

Zeros used to give the correct place value in a rounded number are not significant. If 186,284 is rounded to 186,000, the three zeros are not significant. Zeros are also used before significant digits in a decimal fraction to give correct place value. Zeros used in this way are never significant. If the three-digit measurement 86.3 millimeters is changed to 0.0000863 kilometers, it is evident that the zeros are not significant.

Pupils who secure their own data through measuring will always know which zeros are significant, and no author should ever give a rounded number in a problem without telling the number of significant digits; for example: The speed of light, correct to three digits, is 186,000 miles per second. How long will it take light to go 7,927 miles? Or the speed of light is 186,300 miles per second (4 significant digits), or, using scientific notation, 1.863×10^5 miles per second.

To find how many significant digits there are in a measurement when decimal measures are not used, express the measurement in terms of the *smallest unit of measurement used.* Thus, 46° has two significant digits; 9 inches has one significant digit, but 9 0/64 inches or 576/64 inches has three; 68 pounds 4 ounces or 1,092 ounces has four significant digits; 9 feet has one significant digit, but 9 feet 0 inches or 108 inches has three significant digits; and 9 feet 0/32 inches or 3,456/32 inches has four significant digits.

When changing measurements containing common fractions to decimals, retain only the same number of significant digits as there are in the original data. Thus $6\frac{3}{8}$ inches or $\frac{51}{8}$ inches (2 significant digits) is not equivalent to 6.375 inches (4 significant digits), and $2\frac{2}{3}$ inches or $\frac{8}{3}$ inches (1 significant digit) is certainly not equal to 2.667 inches (4 significant digits). There may be an error of one-

half the unit of measurement in $6\frac{3}{8}$ inches. Since the unit of measurement is $\frac{1}{8}$ inch the error may be as much as $\pm 1/16$ inch or ± 0.06 inch. With the possibility of an error this large, no careful or honest computer would change $6\frac{3}{8}$ inches to 6.375 inches. It must be remembered, however, that abstract hypothetically exact fractions may be carried out to as many digits as is desired. The $\frac{1}{3}$ that appears in the formula for the volume of a cone may be carried out to ten or more digits if this accuracy is desirable. Actually it should be carried out to one more digit than the number of digits in the least accurate measured data with which it is used. Also, all the fractions in the series, $\frac{1}{2}$, $\frac{4}{8}$, 32/64, 64/128, 0.5, and 0.5000000 are equivalent provided they are exact, abstract fractions. If the above fractions are measurements, they are not equivalent.

Rounding Numbers. All elementary texts teach pupils *how* to round numbers, but they usually *do not teach* pupils the really important concepts and practices: *when* to round numbers or *why* the answers to certain problems must be rounded, and *how to determine the number of digits to retain* when rounding data or answers. Some situations in which it is desirable or necessary to round numbers follow: (1) In constructing graphs, it is usual to round data to two, three, or, in very large graphs, four significant digits. (b) Data obtained from handbooks may be more accurate than are needed to work a problem. Thus the value of π to six digits is 3.14159. To find the circumference of a circle 8.6 feet in diameter, π should be rounded to 3.14, or to one digit more than the data. All conversion factors and similar data are rounded in this manner. (c) Data to be learned are usually rounded. Thus the speed of light is usually rounded from 186,284 miles per second to 186,000 miles per second; the diameter of the earth at the equator is usually rounded from 7,926.8 miles to 8,000 miles; the acceleration due to gravity at sea level (latitude 45° is usually rounded from 32.172 feet per second to 32 or 32.2 feet per second; and the length of the light year is usually rounded from 5,875,156,800,000 to 6×10^{12}. (d) Computation usually introduces digits that are not significant. Such spurious digits must be rounded off. (e) If an answer to only n digits is desired, it is customary to round all data to $(n+1)$ digits before computing. Some authorities round to $(n+2)$ digits. Later in this article rules are given to determine the number of digits to be retained in rounded answers.

The following rules may be used for rounding numbers:

Rule 1. If a number, correct to a certain number of significant digits, is to be rounded to a smaller number of significant digits, the digits that are dropped should be replaced by zeros. When the digits that are dropped are located to the right of the decimal point, they should not be replace by zeros. Thus the polar radius of the earth, 3,949 miles, rounded to two digits is 3,900 miles; but 1° of latitude at the poles, 69.41 miles, rounded to two digits is 69 miles.

Rule 2. If the first of the digits that are to be dropped is 5, 6, 7, 8, or 9, the last digit retained should be increased by 1. The mean distance from the earth to the moon, 238,854 miles, rounded to three significant digits is 239,000 miles; and 1° of latitude at the equator, 68.71 miles, rounded to two significant digits is 69 miles. Some texts say that when 5 is dropped, the preceding digit should be increased by 1 if it is odd but left unchanged if it is even. About the only advantage of this rule, if used, is that all students will round in the same way. Since there are as many even as odd digits, when a large number of addends are summed, the errors will tend to compensate.

Adding and Subtracting Approximate Numbers. In a set of measurements like those shown in *a*, each measurement should be given to the same unit; in this case the tenth of a foot is the unit of measurement, since each measurement is made up to the nearest tenth of a foot.

a.	*b.*
42.6 feet	26.24 feet
80.0 "	30.00 "
76.3 "	16.36 "
8.0 "	41.03 "
206.9 feet	113.90 feet

c.
9 8/16 inches
4 3/16 "
5 1/16 "
3 0/16 "
21 12/16 inches

In set *a*, the zeros are significant and must be written as shown. In set *b*, since the unit of measurement is 0.01 foot, the zeros are significant and cannot be omitted. In set *c*, the unit of measurement, 1/16 inch, is clearly indicated. The 8/16 inch should not be reduced to $\frac{1}{2}$ inch. The 0/16 is necessary to show the same unit has been used for each measurement. The 3 0/16 in *c* is just as necessary to show that this measurement is accurate to the nearest 1/16 inch as is the 8.0 feet in *a* to show that that measurement is accurate to the nearest 0.1 foot. There is a vast difference between exact, abstract fractions and the concrete fractions obtained by measuring. The fractions obtained by measuring are never exact and must be treated as all approximate data are treated. The last digit of each measurement in *a* and *b* is a nearest digit and may be slightly too large or slightly too small. For reasons given above, the last digit of the answers to *a*, *b*, and *c* may not be significant, but in practical work the best rule is to retain the full answer. If the 12/16 in the answer to *c* is reduced to $\frac{1}{4}$ inch, the indicated accuracy of the answer will be reduced from three significant digits to two significant digits, the precision of the measurements will be lessened, and an important part of the original data the unit of measurement, will be lost.

In subtracting approximate numbers it is better to retain the full answer, as shown in *d*. Quantities to be added or subtracted should be measured to the same unit. Sometimes it is necessary to add measurements made by different people or measurements and results obtained from computation. In these cases it may be necessary to round the data to the same unit before adding.

d.	*e.*
86.24 inches	251. feet
23.41 "	32.641 "
	4.37 "
62.83 inches	120.5 "

f.
3. inches
1.8762 "

Unfortunately, examples in addition and subtraction like *e* and *f* may still be found in certain textbooks and in examinations containing obsolete processes. The pupil is usually instructed to "fill the empty places with zeros and then add or subtract." Examples of this type do not occur in real life, and they should not appear in up-to-date books. Reeve says, "Not only is an example like 12.7—4.0396 a non-essential, but it is an evidence of educational ignorance" [12].

Writing about similar examples found on so-called standard tests (9.4—4.00083 and 9.2—3.00061), Upton says, "How is a teacher to keep her balance when presumably authoritative tests give problems like the above?" [13]. Brueckner and Grossnickle say in connection with similar examples, "The teacher who gives examples of this type is defeating one of the purposes of teaching decimals. Neither business nor science uses such procedures." And, again, "Ragged decimals never occur in social usage" [5].

It should be noted that if the zeros in an example like *g* are not significant, the example violates the rule that all measurements to be added or subtracted should be measured, or rounded, to the same unit. This is another case of "ragged decimals."

g.
46 feet
210 "
3,400 "
163,000 "

Examples e, f and *g violate the fundamental rule that all measurements to be added or subtracted should be measured to the same unit.* Certainly no experienced

person would think of using four different units in one set of measurements. No one uses a yardstick for a part of a set of measurements and a micrometer for the balance.

Computation never increases the accuracy of the original data. Cullimore says, "The frequent habit of carrying results to a greater number of significant figures than the data warrants comes perilously near to lying with figures" [6]. The teacher who annexes zeros in examples like *e* and *f* and thus makes people believe that rough data are very accurate data is no longer "perilously near"; he has arrived. It is *never permissible to annex zeros* in examples like *e* and *f*. The measurement 251 feet in *e* may be anything between 250.500 feet and 251.500 feet; therefore 251.000 feet is only one of a thousand possible values, each of which would produce a different answer.

A measurement like 3 inches (f) is a very rough measurement, whereas 3.0000 inches is a very precise (small unit) and accurate (five significant digits) measurement. In no sense are 3 inches and 3.0000 inches equivalent. If the 3.0000 inches had been measured correctly to the nearest 0.0001 inch the zeros are significant and no practical measurer would omit them. *Zeros should never be annexed in addition or subtraction.*

Criteria for Accuracy. The measurements 4,832 feet, 48.32 feet, and 0.4832 feet are all equally accurate, since they have the same number of significant digits; but the last measurement is the most precise, since it has a much smaller or more precise unit of measurement. An instrument of precision was needed to make the last measurement. A measurement or the answer to a problem must have the decimal point correctly located, but the location of the decimal point does not indicate the accuracy of a measurement or the accuracy of an answer. Many texts instruct pupils to carry out answers in division to a stated number of decimal places. This is not satisfactory criterion and often leads to serious errors. The only safe criterion is, *carry the answer out to, or round the answer to, the number of significant digits justified by the data.*

The criterion for accuracy that is easiest to apply in the ordinary computation used in elementary schools and in high schools, is the number of significant digits. This criterion is satisfactory for all but the most refined scientific computation. Changing the three-digit measurement 86.3 millimeters to 0.0000863 kilometer does not change the accuracy of the measurement. Since 86.3 millimeters and 0.0000863 kilometer each have three significant digits, they are of the same degree of accuracy. *A measurement having three significant digits, or three-digit accuracy, is more accurate than a measurement having two significant digits, or two-digit accuracy.* An approximate number having four significant digits, or four-digit accuracy, is more accurate than one having three significant digits, or three-digit accuracy. An approximate number having (n+1) significant digits is more accurate than one having only (n) significant digits.

If two measurements have the same number of significant digits, the one that begins with the larger digit is the more accurate. Thus 8.76 feet is more accurate than 342 feet, 4.37 feet, 0.135 feet, or 63.8 feet. The measurement 99.9 feet is very near to four-digit accuracy (99.9+0.1=100.0), whereas the measurement 10.0 feet has just got into the three-digit class (9.9+0.1). If there is an error of 0.05 feet in each of the last two measurements, the error in the first is 0.05 in 99.9 or 1 in 1,998, whereas the error in the second is 0.05 in 10.0 or 1 in only 200. It is easily seen that the error in the second measurement is far more serious than the error in the first. *The ratio of the error in the measurement to the measurement itself is called the relative error.* Where a rigorous criterion for accuracy is needed, the relative error should be used.

The smallest whole number with two

digits is 10, and the largest is 99. Since the unit of measurement is 1, the maximum error to allow two significant digits in each of the above numbers is 0.5. The relative errors will be roughly $0.5/10 = 0.05 = 5$ per cent and $0.5/99 = 0.005 = 0.5$ per cent. That is, in a number with two significant digits the error will range from 5 per cent down to 0.5 per cent. The location of the decimal point will not affect the range of the error. The numbers 99, 9.9, 0.99, and 0.00099 will all have the same relative error.

The range of error for numbers with more than two significant digits may be found in the same way. The table below shows the range of errors for numbers of two to five significant digits.

Number of Significant Digits	Range of Error in Per Cent
2	5 to 0.5
3	0.5 to 0.05
4	0.05 to 0.005
5	0.005 to 0.0005

The measurement 4 0/16 inches or 64/16 inches may have an error of one-half the unit of measurement, or an error of 1/32 inch. Since $1/32 \div 64/16 = 0.0078$ or 0.78 per cent, this falls in the two-digit range and *checks the approximate rule given earlier in this article for finding the number of significant digits when decimal units were not used.* Again, an error of 0.5 ounce in the measurement 12 pounds 4 ounces or 196 ounces will be 0.5 in 196. Since $0.5 \div 196 = 0.0025$ or 0.25 per cent, the measurement has an error between 0.5 per cent and 0.05 per cent and is in the three-digit class.

The expert always writes the full dimension. No designer, draftsman, or engineer would write "2 inches" when he intended a dimension to be 2.0000 inches. It would be neither correct or clear if he so wrote it. In addition to this, the tolerance (the amount of variation permitted in the size of a part) is usually given. If a shaft is to fit in a hole, the dimensions might be given as follows:

hole 2.000 inches $\begin{matrix}+0.001 \text{ inch} \\ -0.000 \text{ inch}\end{matrix}$ Shaft 1.999 inches $\begin{matrix}+0.000 \text{ inch} \\ -0.001 \text{ inch}\end{matrix}$ That is, the hole can vary from 2.000 inches to 2.001 inches, and the shaft can vary from 1.998 inches to 1.999 inches. An engineer would never write 46° for 46° 0″ or 46° 00′ 00″ (either form is permissible). The U. S. Coast and Geodetic Survey goes so far as to write 100° 00′ 00.0″ when a position has been located to the nearest 0.1 second of latitude or longitude.

In contrast to the correct and scientific practice followed by the technicians, the average person very often gives only part of the measurement. Thus a man wanting a table of a certain size, such as 4.00 feet by 6.00 feet, or 4.000 feet by 6.000 feet, or 4 feet 0/8 inch by 6 feet 0/8 inch, or 4 feet 0/64 inch by 6 feet 0/64 inch, will usually write the dimensions 4 feet by 6 feet. It is, of course, impossible to build a table "exactly" 4 feet by 6 feet, but since it is relatively easy to measure to three-digit or four-digit accuracy, a carpenter building the table will most certainly not have an error of 0.5 foot in either dimension. It is reasonable to assume that the error will not be greater than 1/16 inch in such a piece of work by an experienced craftsman.

If the rules for computing with approximate data are followed, the area of a table 4 feet by 6 feet would have to be rounded to one-digit accuracy. The answer 24 square feet would by rounded to 20 square feet. It is obvious, however, that since the intended accuracy is at least that indicated by dimensions of 4.0 feet by 6.0 feet and doubtless by dimensions of 4.00 feet by 6.00 feet and, the answer 24. square feet or even 24.0 square feet is a better answer than 20 square feet. The trouble is not with the rules for computing with approximate data but with the careless way in which the data were given. When our elementary textbooks begin to give their data correctly and to teach pupils how to use these data correctly, we will have far

better mathematics in the elementary schools and far better preparation for high school mathematics and science and for real life applications.

Rules for Multiplying and Dividing Approximate Numbers

Rule 1. *If two approximate numbers have the same number of significant digits, multiply the numbers and round off the product to the same number of digits as there are in each factor. The last digit of the answer will not always be significant, but this rule is satisfactory for all elementary work and is usually followed in scientific work. For example, 26.3 9.25 = 243.275. This result should be rounded to 243.*

Rule 2. *If one of two approximate numbers has more significant digits than the other, first round off the more accurate approximate number so that it contains one more significant digit than the less accurate approximate number. Then multiply the numbers and round off the product to the same number of digits as there are in the less accurate factor. A product can never have more significant digits than there are in the least accurate of the factors used in the computation, and in some cases it may have one less significant digit; but for elementary work the rules given above should be used. For example, 34.5 × 42.1555 = ? should be worked as 34.5 × 42.16 = 1,454.520. This result should be rounded to 1,450. The zero is not significant.*

Rule 3. *If two approximate numbers in division have the same number of significant digits, carry the quotient out to one more digit than is contained in each of the given numbers. Then round off the quotient so that it contains the same number of digits as there are in each of the given numbers. For example, 153 ÷ 62.5 = 2.448. This result should be rounded to 2.45.*

Rule 4. *If the dividend and divisor are such that one of them has more significant digits than the other, first round off the more accurate number so that it contains one more significant digit than the less accurate number. Then divide and carry the quotient out to one more digit than is contained in the less accurate of the two numbers. Finally round off the quotient to the same number of digits as are found in the less accurate number. Thus in the example 884 ÷ 2,150.42, round to 884 ÷ 2,150 = 0.4111; then round the answer to 0.411. Note: 884 ÷ 2,150.42 = 0.41108, which also rounds to 0.441.*

When the above rules for dividing approximate numbers are used, the last digit of the quotient will not always be significant, but these rules are satisfactory for all elementary work and are usually followed in work in science.

The four rules given above may be combined into a single rule. In multiplication and division the answer can never have more significant idigits than there are in the least accurate factor. Answers should be rounded to the same number of digits as there are in the least accurate factor.

A rough demonstration like the one below will help to show the reasonableness of the laws for multiplication. The uncertain digits are in boldface. All digits obtained by using these uncertain digits are also shown in boldface, except those combined with digits carried from the boldface digits. The problem is: What is the area of a rectangle 8.65 feet by 7.43 feet?

$$
\begin{array}{r}
8.65 \\
7.43 \\
\hline
\mathbf{2595} \\
\mathbf{3460} \\
\mathbf{6055} \\
\hline
64.\mathbf{2695}
\end{array}
$$

Best answer 64.3.

It is evident that it would be deceptive to retain more than one uncertain digit. This device may be used for multiplication or division. Pupils may use red pencil for the uncertain digits. A second demonstration is to take the product of the two lower limits (8.645 × 7.425) and the two upper limits (8.655 × 7.435). If all the possible combinations between these limits are taken, there will be 11 × 11 or 121 different answers. Any one of these may be the answer. If however, Rules 1 and 2 are fol-

lowed, the best answer will be obtained in a very large per cent of the cases, and in most of the other cases the error in the last digit of the rounded three-digit answer will seldom be more than 1. In a very few cases it may be 2 or even 3. It is far better to run the risk of a small error in the last digit in a very small per cent of the cases than to follow the present plan of retaining two, three, four, or more digits that are not significant or the equally serious error of dropping digits that are significant.

In the actual work of dividing two approximate numbers, it is sometimes necessary to annex zeros to the dividend in order to secure in the quotient the number of digits warranted by the original data. In the example $86.2 \div 9.43$ the answer should be carried out to four digits and rounded back to three (the answer is 9.14). To secure this answer it is necessary to annex several zeros. If the dividend in such cases is more accurate that the divisor, the original digits may be retained instead of following Rule 4. The zeros annexed in division do not affect the accuracy of the digits retained by following Rules 3 and 4. Four zeros were annexed to get the answer 9.141, which was rounded to 9.14. The same answer may be obtained by rounding to three digits the quotient obtained by dividing any of the following numbers by 9.43: 86.20000, 86.21111, 86.2134, 86.23333, 86.21987, 86.18888, 86.19999, or any one of several hundred other numbers obtained by annexing digits other than zero. In any of the above cases the annexed digits enable the computer to get the number of digits to which he is entitled by the rules that have been given, but they do not change the first three digits of the answer. In actual practice the only digit ever annexed is zero. If this same test is made in addition or subtraction, a large array of different answers will be obtained. It is never permissible to annex zeros in addition or subtraction.

6. In square root the answer should contain the same number of significant digits as there are in the approximate number whose root is sought. The square root of 64 is 8.0, and the square root of 64.00 is 8.000. It is better to carry the answer out one digit farther than warranted and then round back as was done in division. Both the 64 and the 64.00 used above were assumed to be approximate numbers. The square root of an exact number may be carried out to as many digits as desired. In practical work this will usually be determined by the other data in the problem. If the root is to be used with data of (n)-significant digits, it should be rounded to $(n+1)$ significant digits.

7. When used in multiplication, division, and square root, the rules for computing with approximate data should be applied only to the approximate factors. The 4, 3, 1, 2, 6, and 4 in the following formulas are exact numbers:

$$V = 4/3\ r^3;\ A = \tfrac{1}{2}ab;\ A = h/6\ (B+4m+T).$$

Pupils must be trained to differentiate between exact and approximate numbers.

Exact Numbers. 1. Numbers obtained by counting are considered exact. This is especially true when the elements counted are practically identical, such as six nickels, eight 1-inch steel balls, twelve standard eggs of the same grade. When the elements counted are not identical, the "measurement" may be wildly approximate for some purposes. If a new development contains eighty-six houses all built from the same set of plans and costing $7,600 each, the 86 may be considered an exact number. However, if there are in a town eighty-six houses ranging in value from a mansion costing $95,000 down to a shack costing $800, the 86 is exact only in a "census" sense.

Large numbers obtained by counting should be carefully checked to see that counting produced no error. If we read that a certain city has 2,276,385 inhabitants, we may be sure, for a number of good reasons, that little confidence can be placed in the last two or three digits.

2. Small whole numbers in various formulas are almost always exact.

3. Hypothetical measurements may be considered exact. If the sides of a square were exactly 2 inches, the perimeter would be exactly 8 inches, the area exactly 4 square inches, and the diagonal $2\sqrt{2}$ inches. The $\sqrt{2}$ in this case could be carried out to any desired number of significant digits. It might be well to note, however, that no one could construct such a square or measure its sides exactly if it did exist. A hypothetical square foot 12 inches by 12 inches contains exactly 144 square inches, and a hypothetical cubic foot 12 inches by 12 inches by 12 inches contains exactly 1,728 cubic inches.

Approximate Numbers. 1. All measurements of all kinds are approximate.

2. Ratios of measured results are approximate.

3. Many numbers, or ratios, such as $2/3$, π, e, $\sqrt{3}$, tan x, and so on, cannot be expressed exactly by an ordinary mixed decimal or decimal fraction. When the first n digits of such a number are taken as a satisfactory approximation, the number thus obtained is approximate.

4. All rounded numbers are approximate. The answer to any problem in which approximate data are used is approximate and must be correctly rounded.

5. Practically all the numbers taken from various handbook tables are approximate. There are hundreds of such tables, some of which contain thousands of approximate numbers.

6. It is fairly safe to assume that practically all mixed decimals and decimal fractions are approximate.

Miscellaneous Rules and Suggestions. When such approximate numbers as π, 0.7854, 1.732 (the $\sqrt{3}$), e, tan x, or any of the thousands of physical constants, ratios, and reduction or conversion factors found in the various handbooks tables are used in a formula or problem round off the value found in the table to one more digit than the least accurate data in the formula or problem. This agrees with the rules for multiplying and dividing approximate numbers and for partial products. A text should never say $\pi = 3.14$, or $\pi = 3.1416$, or the like. The only satisfactory procedure is to round to at least eight significant digits, making $\pi = 3.1415927$. The pupil should then be taught to round this value to one figure more than his data. Eight significant digits were chosen because an eight-significant-digit measurement is the most accurate measurement that has been made up to the present time. The value of could be given, of course, to six, seven, eight, or as many digits as are desired by the author the value given should contain more digits that any of the data given in the text. The same thing should be done with conversion factors, that is, 1 kilometer = 0.62136995 mile, 1 liter = 1.0566818 liquid quarts, 1 kilogram = 2.2046223 pounds, and so on. The value given should always be sufficiently accurate for any measurement that will ever be needed in the text.

When finding the products of three or more approximate numbers or when squaring or cubing an approximate number, follow the rules for multiplication. If possible use the more accurate factors first. Somewhat better results will be obtained if one more digit is retained in the partial products than will be retained in the final answer. A similar rule may be used in division or an example in which both multiplication and division are needed.

In this vital matter of computing with approximate data in the elementary schools and the high schools we need simple consistent rules that can be easily applied. Statistical and other refinements are matters for the college and graduate school. The rules given in this article form a safe foundation on which the graduate school may build. It is true that when the rules given in this article are used, now and then a final digit that is not significant will be retained. This is not serious and will not conflict with work in science where it is common practice to retain a final figure that is on the "ragged edge."

Many of our modern textbooks are highly inconsistent in their treatment of

computation with approximate data. Duane Roller says, "Most textbooks discuss the concept of significant figures (digits) but fail to take it into account in stating problems of a quantitative nature thus making it almost impossible for the student to use the idea of significant figures in solving problems" [9]. A few modern texts give a fairly good treatment of computation with approximate data, but in succeeding chapters, and in their answer books they fail to follow the rules that they have developed. In other texts the chapter treating approximate data is placed at the end of the text.

The data in any given problem in mathematics or science should be consistent and should clearly indicate the accuracy desired in the answer. No answer book should give, and no teacher should allow, more digits in a final answer in multiplication, division, or square root than there are in the least accurate item. The data in a problem may be approximate but the computation is not. Computation with approximate data is not rough, careless, or slipshod work. It is careful, intelligent computation that produces honest answers. The answers obtained are the best answers that can be secured from the given data.

The words *precise*, *accurate*, and *correct* should not be confused. Precision and accuracy are relative. The measurements 0.00064 inch and 2.43126 inch are equally precise (an equally small unit has been used), but the second measurement is far more accurate (it has more significant digits). If an answer is correct, no mistakes in computation have been made and finally it has been correctly rounded.

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ON TO BALTIMORE!

Outgrowth of a Philosophical Approach to the Teaching of Mathematics

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THIS program was the outgrowth of special emphasis in the teaching of mathematics in my classes in the National Cathedral School. During the last six weeks of the fourth year the pupils' home work was devoted entirely to reading mathematics history. (Most of the classwork was confined to textbook review.) Each girl was given two assignments—one covering the general outline of history and another covering either a special division of mathematics or the life of a prominent mathematician.

The following dialogue was written by the students at the suggestion and under the direction of the teacher and was presented before the school in *May, 1946*. The sophomore who sat in the center of the group and asked the questions had no part in the preparation of the program.

References to the main theme of the dialogue were frequently made by the teacher during the first three years of high school mathematics, but no definite work in the history of mathematics was done until the senior year. Not until the students had a thorough knowledge of the principles of elementary mathematics and a facility in the application of them did they do outside reading.

Ellen. Have you ever really stopped to think why mathematics is taught in every school all over the world? Have you ever wondered what this world would be like without mathematics? Modern bridges and buildings couldn't be built without the help of arithmetic. Housewives and grocers couldn't exchange commodities for money without knowledge of it. Calculating the exact amount of food for a large group of people would be impossible for one who couldn't count. Yet, with all the daily uses for mathematics in this modern world, many educators are no more advanced in their knowledge of mathematics than were the ancients. The early mathematicians, with all their handicaps, probably knew more than the average college mathematics major who has all the theorems proved and simplified for his own convenience. Mathematics, as much as any other science of nature, has been, through the ages, a magical field of exploration. The questions suggested by the study of this subject are varied and interesting. Any high school sophomore might have many of them.

Gay. Why did mathematics originate?

Garland. Mathematics grew out of a need for a method of expressing numerical thoughts. The most fundamental need was the need for counting.

Gay. How did man first attempt to count?

Garland. This was done in the beginning by the use of the fingers. Man learned quickly that two straight marks could be used to represent two of anything. He represented higher numbers by using fingers and toes. The American Indians used the figure of a hand for 5; the Babylonian symbol for 10 may have been derived from two hands.

Gay. Is that why 10 is the base of our number system?

Garland. Yes. If man had not had 10 fingers, it is probable that we should not be using a decimal system. A number system having 12 as a base might be simpler, and if man had been born with 12 fingers rather than 10, it is likely we would use this duodecimal system. Some mathematicians maintain it would be much more feasible because several fractional parts of 12, such as $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ could be found evenly.

Gay. Where did our number system originate?

Garland. The notation for our number system, "called the Arabic or Hindu-Arabic, originated with the Hindus and was borrowed by the Arabs who introduced it into Spain in the eighth century at the time of the Moorish conquest." Hindu mathematicians, though probably influenced by the Greeks at an early period and by the Chinese at a later one, made original contributions themselves.

Gay. What special contributions did the Hindus make?

Garland. The numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 are "directly traceable to nine Sanskrit characters" employed by the ancient Hindu mathematicians. Their system combined simplicity and position value—characteristics which have enabled the modern world to develop a far more advanced mathematical science than that of the ancients. Perhaps the Hindus' greatest contribution was a symbol for zero. The development of mathematics was retarded long after the *idea* of zero had been accepted, because there was no convenient *symbol* for it. The Hindus gave us the zero symbol that we use now.

Gay. I presume the Roman numerals were first invented and used by the Romans.

Garland. Yes, but no real advance in the theory of mathematics is found anywhere in Roman history. In fact the development of mathematics was retarded because the Roman numerals were too cumbersome for performing addition, subtraction, multiplication, and division.

Gay. Which ancient civilization was responsible for our method of writing fractions?

Garland. As early as the seventh century, the Hindu, Brahmagupta, wrote the numerator above the denominator, but the Arabs improved this by placing a bar between. Work with fractions for all the ancient civilizations was roundabout in method. The Egyptians and Greeks had very clumsy methods of notation and manipulation. In Greek arithmetic, fractions were considered as the ratio of two numbers. In the Egyptian mathematics a fraction was simply a part of a number, and this concept of fraction became the most popular. Everyday fractions used to be called "the usual small divisions"; later the name came to be the English "vulgar fractions"; hence the American expression "common fractions." Although the idea of inverting the divisor was known to the Hindus and Arabs, it did not come into common use in Europe until the 17th century. The development of a symbolism for decimal fractions was slow. In 1585 a very complete essay on decimals was written by Simon Stevin, who said the system had been applied and adopted by business men who considered it far superior to their own. The Englishman who published the first translation of Stevin's work was so much impressed with the value of the decimal system that he offered, for the good of the country, to give instruction in it free of charge.

Gay. When did arithmetic become a regular school subject?

Garland. This happened comparatively late in the development of mathematics. During the 12th century universities sprang up all over Europe and in these schools the seven liberal arts—grammar, rhetoric, logic, *arithmetic*, astronomy, *geometry* and music— were taught. Prior to this, the study of mathematics had been confined to the religious orders and leisure class. In the *17th* century arithmetic and geometry were required for *graduation* from Harvard, and not until the middle of the *18th* century did arithmetic in America become a college entrance subject. About one hundred years later *geometry* was first required for admission to Harvard, and has been considered a secondary school subject in this country for only about seventy-five years.

Gay. I gather from what you've said that the growth of arithmetic was retarded by the development of a convenient notation for expressing the relationships be-

tween quantities. Since algebra is even more symbolic than arithmetic, was its development equally slow?

Eleanor. Yes, there are three phases through which we have finally attained the symbolic forms and expressions, such as $x+y=3$, $x^2-5x+5=0$, which we use now in our albegra courses. The following description is given in Dr. Logsdon's book "A Mathematician Explains."

"In the first stage algebra can be called rhetorical, because, lacking any kind of symbol, the calculations are effected by words.

In its second stage algebra can be called syncopated, because it uses words interspersed here and there with abbreviations to make simpler and more rapid the progress of the reasoning and of the calculations.

In the last—and, we may feel safe in saying, almost perfect—stage of development algebra can be called symbolic, because it uses special symbols to indicate the given and the unknown quantities and it also represents the various operations by operational symbols. Words have entirely disappeared from the steps of an algebraic proof or solution, though they may be used in explaining the process. Not until the seventeenth century did algebra reach a stage nearing that of perfection as regards symbolic representation."

Gay. I've always heard that mathematics began with the dawn of civilization, and for that reason it seems strange to me that these simple symbols we easily understand and use in first-year algebra were not invented until a few hundred years ago.

Eleanor. That is not surprising when we know that long division was not generally known when Columbus discovered America.

Gay. Were there any mathematicians who were outstanding in their contribution to the *third* phase of algebra development?

Eleanor. Yes, François Vieta, a French lawyer and politician, who had many publications on algebra and trigonometry was one of the first to make an extensive use of letters to represent numbers. Descartes was the mathematician who introduced the last letters of the alphabet x, y, and z to indicate variables or unknowns and the first letters of the alphabet to indicate constants.

Gay. If those simple algebraic symbols weren't used until almost 400 years ago, when did mathematicians begin to use exponents?

Eleanor. Though Vieta in the 16th century introduced the type of algebra we know now, mathematicians of his day wrote x square and x cube instead of x^2 and x^3, and as late as the 18th century American students used the notation xx and xxx. The *idea* of exponents was not new to mathematicians who preceded Descartes, but to him we owe our present notation for writing them. About thirty years later Wallis and Newton, two great mathematicians, used fractional and negative exponents.

Gay. When was the algebra that we study today as freshmen and sophomores first taught in the secondary schools in this country?

Eleanor. In American universities elementary algebra was taught as a college subject until the middle of the 19th century, so it is possible that our great-grandfathers learned their Algebra I and Algebra II in college.

Gay. I take it that there were no math college boards in those days.

Eleanor. Well, if there were entrance examinations for college, they were in arithmetic, not algebra.

Gay. You have named *French* mathematicians who contributed much to the third phase of algebra development. What ancient civilizations produced mathematicians who were famous algebraists? Weren't the Greeks of classic times interested in algebra?

Eleanor. Not especially; there was one famous Greek, Diophantus, who compiled all the algebraic knowledge of the Greeks

in a work called "Arithmetica." This was during the 3rd century. Much of the work in "Arithmetica" was contributed by other mathematicians but some of the problems and ideas were original with Diophantus. According to one historian, "he used symbols for an unknown quantity and its powers; he wrote quantities that were to be added side by side, but used a symbol for subtraction; he knew the rules of signs for multiplication, but classed the negative, irrational, and imaginary roots of an equation as *impossible*." Much of his work included verbal problems of the "find a number" group whose solutions were difficult if not impossible to find. Hypatia, the earliest of the few women mathematicians that we read about, wrote commentaries on "Arithmetica."

Gay. It seems to me that I've heard of Hypatia before, but not in connection with mathematics.

Ellen. Yes, you have heard it in English literature. This same Hypatia is the principal character in a novel by Charles Kingsley.

Eleanor. Records have been kept of the algebraic works of four *Hindu* mathematicians—Aryabhata, Brahmagupta, Mahavira, and Bhaskara who were especially interested in series, permutations, verbal problems, and equations. One method of solving the quadratic equation is known as the Hindu Method. However, from about 800 A.D. until the time of the Crusades, Arab mathematicians, especially interested in quadratic and cubic equation solutions, were among the foremost algebraists of the world. The only one whose name would be familiar to you is the Persian poet, Omar Khayyám. He solved thirteen different types of cubic equations but failed to give a solution of the general case because his number system comprised only positive integers and fractions. The most famous Arab mathematician was Mohammed ibn Musa al-Khowârizmi, who wrote *Al-jabr w'al muq â balah*. This treatise means "restoration and reduction" and deals with transposition and collection of similar terms. The title of this work gave us the name Algebra.

Gay. In second-year algebra we solve quadratic equations with negative, irrational, and imaginary roots. You said Diophantus considered these solutions impossible. Was it difficult for mathematicians to acquire these new number concepts and invent a suitable notation for them?

Eleanor. The acquisition of new concepts is usually slow and hard. Not until the close of the 16th century were negative numbers no longer considered absurd. Hudde was the first to let a letter represent either a positive or a negative number "thus permitting a single rule to cover many special cases." In finding the approximate values of irrationals, that is, finding the square root of imperfect squares, one important method that we learn in school today is due to Heron of Alexandria. There are several methods which were discovered as early as 225 B.C. and as late as 1140, but the solution of equations with irrational roots was not used at all generally until the late 16th century. As late as the *18*th century imaginary roots of an equation were being discussed by the Paris Academy but by 1800 mathematicians could find approximate values of the roots of algebraic equations with numerical coefficients.

Gay. I've heard seniors talk about trigonometry. What is trigonometry?

Frances. The word trigonometry comes from two Greek words meaning "triangle measuring," and in the beginning it dealt solely with finding distances to inaccessible objects by considering them sides of triangles, other parts of which could be found. However, the invention of calculus led to the development of a branch of the subject known as "Trigonometric Analysis" that has many practical applications in calculus and higher analysis.

Margaret. You asked about the Greeks a while ago in reference to their algebraists. As a matter of fact they were noted for their geometry, and some of the outstand-

ing ones in this field were Thales, Pythagoras, Plato, Aristotle, and Euclid.

It was necessary in the growth of civilization for men to measure lines and areas. The Egyptians used practical geometry in order to construct the exact proportions of their pyramids and to survey the Nile Valley after the river had overflowed its banks. This mathematical knowledge was transmitted from the Egyptians to the people who lived around the coast of the Aegean Sea. The most famous of these men was Thales. He made geometry a deductive science and was responsible for the first logical proofs in mathematics. Thales lived from about 640 to about 546 B.C. in Miletus, a city on the coast of Asia Minor. He was one of those fortunate people who had amassed a large enough fortune in shrewd business dealings to enable him to turn to the study of mathematics and philosophy as his lifework. He traveled much, was fascinated by the pyramids, and measured their height by shadow reckoning. The most noted pupil of Thales was Pythagoras, who lived during the 6th century B.C. He is supposed to have founded a secret society of wealthy and scholarly men, known as the Pythagoreans who studied arithmetic, music, geometry, and astronomy. They are said to be the authors of several of the theorems that we study in geometry today.

Gay. Haven't we used one of Pythagoras' formulas in second-year algebra?

Margaret. Yes; $a^2+b^2-c^2$, the theorem known as the Pythagorean theorem, the square on the hypotenuse of a right triangle equals the sum of the squares on the other two sides; and according to tradition he sacrificed an ox on discovering the proof.

Plato and Aristotle, famous Greek philosophers, lived about 400 B.C. and are considered mathematicians because of their attitude toward and interest in that subject, rather than because of their original contributions. Plato maintained that geometry was a valuable mental discipline, and above the entrance to his academy was the inscription, "Let no one ignorant of geometry enter here." The study of mathematics influenced Aristotle's system of logic and he urged his students to do research in the historical aspect of the subject.

One cannot speak of Greek geometry without mentioning Euclid, who about 300 B.C. summarized all the mathematics known in his time and included it in one book known as the "Elements."

Gay. I thought that Euclid's "Elements" contained only geometry.

Margaret. Naturally you did, because the geometry that we study as high school Juniors is taken directly from the "Elements" and is called Euclidean geometry. However, contrary to popular thought, Euclid's "Elements" contained all the mathematical knowledge of his time and included work on number theory and irrationals as well.

Gay. Were there any stumbling blocks for the Greeks? Were there any problems that puzzled them?

Frances. Yes, there are three problems known as "the three famous problems of antiquity"—the trisection of the angle, the squaring of a circle, and the duplication of a cube. They are ruler and compass constructions. It is only in recent times that it has been shown that it is not possible to solve them by use of the straight edge and compasses alone. They are to be constructed by compasses with which we draw a circle and by a ruler with which we draw straight lines without measuring lengths. The instruments are to be used a finite number of times, and the construction depending on Euclid's axioms and theorems is to be absolutely correct. If we give up these requirements, the problem becomes possible.

The problem of duplicating a cube originated when the oracle at Delphi was consulted in regard to a means of placating the gods who had inflicted a plague on a tribe in ancient Greece. The oracle assured the people that the plague would be stopped if the golden cubical altar in the

Temple of Apollo were replaced by another containing twice as much gold. The masons and architects made the mistake of doubling the side of a cube, thus making the volume eight times as great. The oracle was not satisfied, so Greek mathematicians began to see that the right answer involved not doubling a side but rather multiplying the side by the cube root of two. This could not be done geometrically with a straight edge and compasses. They succeeded by using other instruments and higher curves, and the epidemic ceased.

To square a circle means that we should construct a square whose area is equal to the area of a given circle by using straight edge and compasses, as explained before. It was in 1882 that all hopes of performing this feat were removed when a German mathematician, Lindemann, published a proof that the squaring was impossible. It can not be performed because π, the ratio between the circumference and diameter, is transcendental. For this reason people who attempt the impossible are called "circle squarers." Many persons, however, are still attempting to find solutions to these problems.

Gay. By this do I understand you to say that it is impossible with straight edge and compasses to trisect an angle or to construct a square equal in area to a given circle?

Frances. Yes.

Gay. But doesn't one occasionally see published so-called proofs of angle trisections in the daily papers and magazines?

Frances. Yes, but those are very often written by bright high school boys interested in construction who are ignorant of the fact that mathematicians have proved them impossible. All recent solutions have been incorrect, and the error committed is usually one of these four:

1. The solution is merely approximate instead of exact.
2. Instruments other than straight edges and compasses are used.
3. There is a logical fallacy in the pretended proof.
4. Only special angles such as 90° or 180° instead of an arbitrary or general angle are considered.

Gay. Was there any Greek mathematician more famous than Euclid?

Ellen. Yes. Archimedes, one of the greatest mathematicians of all times, contributed a great deal to the field of solid geometry. Like Euclid, Archimedes wrote on all the mathematical subjects then known, but unlike Euclid, he produced much original work instead of a compilation of the works of others. The work for which he is best known is that of the mechanics of solids and fluids, though his geometrical discoveries about the parabola and the spherical surface and his rule for finding the volume of a sphere were remarkable. It was his seemingly miraculous feats with mechanics that made him popular with the common people. Archimedes thought it disgraceful for a philosopher to apply the results of science to practical use and was ashamed of these works. Yet when there was dire need of mechanical assistance, Archimedes produced another practical invention. It has been said that if the Greek mathematicians and scientists had followed Archimedes instead of Euclid, Plato, and Aristotle, they could have easily anticipated the age of modern mathematics and the age of modern physical science by 2,000 years. Some even say that if Archimedes were alive today, after taking one post-graduate course, he could talk intelligently with Einstein and other modern mathematicians.

There are many stories of how, when Syracuse was besieged by hostile ships, he set fire to them by the aid of mirrors arranged to concentrate the sun's rays upon them. He may have startled many in his time by his boast "Give me a place to stand, and I will move the earth," but none questioned it, for he had proved that by the aid of one compound pulley one man could launch a heavy-laden ship with very little exertion. One of the most well-known stories told of him is possibly legendary but interesting. King Hiero, a

friend of Archimedes, had given him the task of proving his crown all gold without cutting it open. While in the public bath one day, he watched how the weight of his body raised the water. Suddenly the thought struck him. He jumped from the bath and rushed out into the streets shouting "Eureka—I have found it." With this discovery he proved that the weight of water displaced by the crown was equal to that displaced by a brick of equal weight in gold.

The one achievement of which Archimedes was proudest was the finding of the volume of a sphere in relation to the volume of its surrounding cylinder. He was so fond of the symbol that he asked that it be placed on his tombstone.

Archimedes' life came to an end when the Roman general, Marcellus, captured Syracuse. Accounts of his death are legendary. According to one story, Archimedes was drawing diagrams in the sand when a soldier stepped on them. Upon his objection, the soldier stabbed him. Marcellus gave him an honorary burial. His tomb was unknown for many years, but Cicero discovered it by recognizing the symbol of a sphere and cylinder upon the stone.

Gay. Do you mean that Archimedes was so interested in his mathematics that he drew diagrams completely unaware of the battle and danger going on around him?

Ellen. According to the story, that is true, and there's a famous picture of it taken from a mosaic found not long ago in the ruins of Pompeii.

Gay. Archimedes must have been peculiar.

Ellen. No, not peculiar, but unusual. Newspapers published a case of similar detachment during the past war. The philosopher, George Santayana, was found living in a small Catholic institution in Rome. When questioned by United States Army officers who were interested in his opinion of the time, he said: "I live in the eternal." Of war he knew nothing.

Gay. What was the name of the man who was the first to use x, y and z to denote unknowns in algebra?

Eleanor. Descartes.

Gay. Was he a famous mathematician?

Bettie. Yes, a great creative mathematician and philosopher of the 17th century, an inventor of analytic geometry, a gentleman and a soldier who never taught a class or studied under a famous teacher.

Gay. What is analytic geometry?

Bettie. Analytic geometry is geometry expressed algebraically—the union of algebra and geometry. This mathematics consists in associating numbers with points, and equations with geometrical figures. It is the study of the conic sections—the circle, ellipse, parabola and hyperbola, and their second-degree equations, as $x^2+y^2=r^2$ for the circle and $y^2=4ax$ for the parabola.[1]

Gay. We've had that in algebra.

Bettie. You do have a very elementary touch of it in your *second-year algebra* and the fundamental notions of it you first use in drawing the graph of a straight line in *first-year algebra.* However, you study and graph equations of only those conic sections whose centers are at the origin and you never develop and use the analytic geometry formulas for finding the distance between two points, the midpoint of a line segment, the slope of a line, or the tangent to a curve at any given point.

Gay. The person who first thought of associating a pair of numbers with a point on a plane must have had a great deal of imagination.

Bettie. Yes, that's why he's called a *creative* mathematician. He introduced into mathematics (as Einstein has done in the field of mathematical physics) an entirely new concept or way of thinking about the real number system and the points in a plane.

Gay. I wonder how he happened to think of it.

Bettie. Some say that he *first* thought of

[1] It would be more correct to say: In this geometry conic sections are studied by means of equations.

it while he was lying in bed and saw a fly crawling near the corner of the ceiling, and there is another story to the effect that one night it was revealed to him in a dream. However, it is obvious that the mathematical accomplishments of the preceding time made the development of such a method of connecting algebra and geometry imminent, for Fermat, another Frenchman, invented an analytic geometry at about the same time.

Gay. Are there similar cases in the history of mathematics where branches of the subject have been developed and published by two different mathematicians without each knowing of the other's work?

Roxanne. Yes. A quarrel was started in the seventeenth century when both Gottfried Leibnitz, a German, and Isaac Newton, an Englishman, claimed to be the inventors of the calculus. They published their works at about the same time, and their results were quite similar. Instead of perfecting the notation Newton used to express the ideas of calculus, the Englishmen kept up the national argument to determine who should have the credit for its creation. Liebnitz' method was further developed by his followers so that ultimately, no matter who was responsible for the invention of calculus, the German method of notation was superior to the English.

Gay. Weren't the mathematicians foolish to continue this argument about who invented calculus?

Roxanne. Yes. This step in the development of mathematics was inevitably next. It was easily possible for two great mathematicians to create the same ideas at approximately the same time. Newton, himself, stated this when he said, "If I have seen a little farther than others it is because I have stood on the shoulders of giants." Both Leibnitz and Newton had the framework for their discoveries built by mathematicians preceding them and both, therefore, could make the same conclusions.

Gay. Isn't calculus a college subject? What is it?

Roxanne. Yes. *Differential* calculus pertains to the rate of speed of objects in continual motion. It makes use of their velocity or change of distance with respect to time and acceleration or change of velocity with respect to time. The binomial theorem that you become acquainted with in second-year algebra is basically essential. You find $(a+b)^n$ when "n" has integral values; in calculus "n" is given *fractional* and *negative* values. You use such terms as variables, functions, and limit, the basic concepts of which we first use in second-year algebra. *Integral* calculus is concerned chiefly with integration, a process enabling us to find, among many other things, the area of a surface of revolution and the volume of a solid of revolution.

Gay. Is there any mathematics known besides Einstein's theories which has been so recently conceived that it isn't yet taught in college, as calculus wasn't many years ago?

Day. Non-Euclidian geometry, which was formulated by Lobachevsky, a Russian, at the beginning of the last century, is based on concepts so different from any we are accustomed to that they are nearly as difficult to imagine as the ideas on which the atomic bomb is based.

Gay. What are these ideas and are they really so difficult?

Day. The idea in non-Euclidian geometry which makes it different from Euclid's plane geometry is that there can be more than one straight line in one plane parallel to another through a point. This is really not so difficult to understand if you are willing to admit that there can be two ideas of what a straight line really is.

Gay. What are these two ideas?

Day. The definition of a straight line is the shortest distance between two points. On a sphere the shortest distance between two points, as all sailors know, is not what you usually think of as a straight line. In the plane surface forming the surface of a sphere there can be *no* parallel lines if you admit that parallel lines are formed by two

straight (remember the definition) lines which never meet and are always equidistant from each other. In the plane surface forming the surface of a pseudosphere[2] there can be two lines through a point parallel to another line. These ideas are not difficult, just foreign to our everyday thought.

Gay. How did anyone come to think of these ideas?

Day. For many years people had been trying to prove by Euclid's theorems his postulate stating that "through a point in a plane there can be drawn only one parallel to another line in the plane." No one had succeeded. Lobachevsky decided that another set of theorems, different, but nearer abstract truth, could be built up by assuming the "parallel postulate" false.[3] He was the first to work this conception out logically and published a book on it when he was only 30 years old. It made him famous but he did it neither for fame nor to answer a material need or application. He did it because he wanted to get nearer abstract truth. Lobachevsky also wrote many other pamphlets, books, and teaching manuals in his 33 years as professor or as director of the University of Kazan, the first position he held before he was 30.

Gay. How did Lobachevsky when he was so young get such a high position in rigid Tsarist Russia?

Day. It was even more amazing when you realize that he was the son of a serf. The village schoolteacher and priest saw his genius and got him a scholarship at one of the Tsar's schools, where he soon took high honors. It was his creative genius that made him so outstanding and procured for him his high position.

Gay. Bettie said that Descartes never taught a class in mathematics. Were few of the great mathematicians teachers?

Margaret. Euclid taught at the University of Alexandria in Egypt.

Day. Lobachevsky taught at the University of Kazan in Russia.

Frances. Sylvester taught mathematics at the Royal Military Academy, Woolwich, England and in 1876 came to America to become the first head of the department of mathematics at The Johns Hopkins University in Baltimore.

Roxanne. Isaac Newton became professor of mathematics at Cambridge when he was 27 years old and taught there for 26 years. Leibnitz was a private teacher to the Electress of Brandenburg and founded the Berlin Academy of Sciences, which existed in Germany until the last war. However, Leibnitz was a "master of all trades"—a genius in the study of mathematics, law, religion, statecraft, history, literature and logic. He produced something outstanding in all of these fields, and some biographers think that if he had not divided his interest but concentrated all his time and effort on mathematics, he would have been the greatest mathematician of all time. Newton, though not as versatile as Leibnitz, was a very great scientist and student of theology. The "Principia" which contains accounts of his astronomical and dynamical discoveries, is considered the greatest book of its kind ever published. Some think it a great shame that Newton resigned his professorship at Cambridge to accept the post of Warden of the Mint. He later became Master of the Mint and was knighted by the Queen, because, as one historian expressed it, he thought it was more important to be called "*Sir*" than "*Mr.*"

Bettie. Descartes never taught a class but he made a tragic attempt to instruct a private pupil. It seems that the energetic queen of Sweden, nineteen-year old Christina, having vigorously invaded all other fields, decided to pursue the study of mathematics. Since Descartes was then the leading mathematician, he was called to the Swedish court. Although he did

[2] For a definition of *pseudosphere* see *The Three Moons in Mathesis* by Hugh and Lillian Lieber.

[3] It would be more correct to say: Lobachevsky decided that another set of theorems could be built up by substituting a different postulate for Euclid's postulate of parellels.

not wish to leave his quiet cottage in Holland, he was finally persuaded to go. The young queen was extremely busy and had scheduled her mathematics class for five o'clock in the morning. Now, Descartes had always been a sickly person and all his life had remained in bed until eleven o'clock, even when he was at boarding school. As a matter of fact, it was during these leisurely mornings that Descartes did most of his scholarly philosophising. The rigorous demands of this domineering queen and the sharp climate of her country proved to be too much for Descartes. He caught cold and died. This is a case in which the *pupil* killed the *teacher* instead of the usual reverse.

SUMMARY

Roxanne. By reading several histories of mathematics an entirely new approach to the subject has been given *us.* We have found the famous mathematicians to be interesting human beings instead of mere words on a page or names spoken by only those who had extensive training; we have acquired a general understanding of subjects we shall study in college, and we have found a cultural background for further reading.

Day. It is easy to see that as your fundamental notions change, the final truths at which you arrive change. While the ancients believed that irrational and negative numbers did not exist, they were unable to solve many equations. As long as lines were thought to possess only the quality of length, and as long as it was assumed that through a given point only one line could be drawn parallel to a given line, there was *one* instead of *many* geometries. Deductions and conclusions are determined by fundamental concepts and final assumptions The study of mathematics makes this unique cultural and practical contribution to our secondary school education, for in order to penetrate thoroughly such college subjects as science, economics, and religion, and to understand completely certain phases of our life, we must recognize and apply this mathematical process.

Ellen. Now we see that mathematics is not only a system of using numbers, but a great and stimulating challenge to all penetrating minds. It is one of the purest of the sciences and the different conceptions of it have opened wide fields of experiment to those with great imagination. The broad scope and wide range indicate that so far the creative mathematicians have merely scratched the surface.

Gay. I certainly want to thank you seniors for this enlightening discussion. I feel very grateful for the inheritance of several centuries of work of great mathematicians, but I also feel very humble when I realize that the mathematics I know doesn't extend beyond the 16th or 17th century.

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Are We Giving Our Mathematics Students a Square Deal?

By VERYL SCHULT

Head of Dept. of Mathematics, Divs. 1–9, Washington, D. C. Public Schools

A GREAT DEAL is heard these days about the peacetime draft. However, another draft has been under way—the draft of America's children. A huge peacetime army has again been conscripted into the nation's schools this year to be trained for the greatest of all wars—the war against ignorance. What responsibilities have we as mathematics teachers? After requiring the students to be in school, and requiring them to take certain work in mathematics, are we giving them a square deal? I am not going to answer that question with a YES or NO. I will just present the evidence and let everyone decide for himself. One way to decide would be to answer questions such as these:

I. ARE WE DOING RIGHT BY THE TOP 25 PER CENT OF OUR STUDENTS— THOSE WITH HIGH IQ'S

Nicholas Murray Butler once spoke of America as the "best half-educated country in the world." Isn't one reason for that the fact that we have "watered down" so much of our teaching to adjust it to mediocre students?

When the American Association for the Advancement of Science met in Washington for its Centennial Celebration, President Truman said in his address which opened the meetings, "The secrets of nature are not our monopoly. Any nation that is willing and able to make the effort can learn the secrets that we have learned" And then he emphasized the necessity of our training men and women who can do this needed research. Valuable guidance is given in the Report of the President's Scientific Research Board, commonly referred to as the Steelman report. Volume 4 of that report, MANPOWER FOR RESEARCH, discussed the training recommended for the top 25 per cent of our students in science and mathematics, since that is the group that is needed to be leaders in science. This report is available from the Superintendent of Documents, Government Printing Office, Washington, D. C., for 35 cents. "This is the age of science," says the Report; and it continues, "The United States now holds the scientific leadership of the world. However it is a question, whether, even in the United States, there is a clear understanding of what science can do for mankind. Therefore, we must not only develop a corps of scientists sufficient in number and competence to insure continued progress but also make a concerted effort for mass education of our citizens in scientific principles and attitudes." It quotes our shortcomings in the famous Nimitz letter and suggests that the "real Pearl Harbor" was not out in the Pacific but in the mathematics classrooms of our schools. Then it makes comments such as the following about mathematics:

1. Many unsatisfactory solutions to the problems of ninth grade mathematics have been attempted. To meet the present situation, many schools now provide a double track in mathematics—algebra for those whose ability and future outlook indicate clearly that they should take it and a good course in general mathematics for the rest.
2. Good guidance is necessary. "The ideal of a strong high school mathematics department should be to have every pupil in the appropriate course with no one dissatisfied. In guiding the pupil into the appropriate course, ability tests of reading, intelligence, and computation are very helpful . . . ."
3. Guidance services for curriculum revision should be provided at the local, state, and national level.
4. In recent years, many schools have not scrutinized their traditional courses in the light of recommendations such as the 1923 Report and the 15th yearbook of

the National Council of Teachers of Mathematics, and discarded the obsolete and brought in the new. The Report criticizes the schools for sending men with good native ability into the Army and Navy with no clear understanding of such important concepts as vector, tolerance, interpolation, representative fraction, scale drawing, tangent of an angle, micrometer, vernier, gage block, metric system, practical constructions, logarithms, and the slide rule. "Yet," says the Report, "These are among the very things that would have added vitality and meaning to the abstract symbolism and theory which their former mathematics teachers struggled to teach them."

5. If courses are kept rigorous for the better students, that means providing good courses for the other students. People in small schools may see difficulties in offering a variety of courses. The Report Recommends for small schools:
   a. Offering two courses simultaneously in the same class period. Experiments find it feasible, and rural school teachers always taught six to eight classes simultaneously.
   b. Correspondence courses in algebra, geometry, and other secondary mathematics courses are available. For instance, the State of Wisconsin has authorized Boards of Education to buy such courses for pupils.
6. Closer correlation between mathematics and science is recommended.

II. Are We Using Our Rich Government Resources?

The Washington, D. C. mathematics teachers recently visited the Bureau of Standards to learn about things which would enrich mathematics teaching. They were taken into the vault where the present standards of measure are kept. They visited the Applied Mathematics Laboratory and learned about the work there; they saw the remarkable sundial which is worth a trip there just to see, and also the beginnings of the remarkable new electronic computing machine. The Bureau of Standards is just one of many Bureaus which are doing work which could be utilized in enriching our teaching of mathematics.

The Civil Aeronautics Administration is conducting an experiment this year to find how mathematics teachers can utilize aviation materials which are so exciting to students. At the end of this school year the experiment will be written up and it can be obtained from the Civil Aeronautics Administration. A very helpful aviation reference is the *Aviation Education Source Book* published by Hastings House ($8 less 25% to schools). This book contains several hundred pages of practical applications of mathematics to aviation problems, and would be a valuable addition to a teacher's library. The C. A. A. has other helpful teaching materials which are available for the asking.

The Education Section of the School Savings Division of the United States Treasury keeps a supply of good materials coming to the schools. Several of these, which are available in quantities for mathematics classes are:

Teaching Mathematics Through School Savings
Budgeting for Security
Lessons in Arithmetic Through School Savings

The Women's Bureau of the Department of Labor has published a pamphlet, *The Outlook for Women in Mathematics and Statistics*, which is very helpful in guidance in mathematics.

In Washington, the Federal Government is our "community," but that leads me to my next question.

III. Are We Making Good Use of Our Own Community to Enrich our Teaching of Mathematics?

The comment has been made that schools should be *a part of* not *apart from* the community. A questionnaire was sent to our mathematics teachers last year to find out how they were using community resources to enrich their teaching. It revealed many possibilities which one teacher alone probably would not think of:

1. Many classes used community data from drives such as Community Chest and Red Cross, the city budget, school data sent out by the statistics department, and the current water, electricity, and gas rates, in problems and in graph making.

2. Some classes used blueprints of their school in their study of measurement and scale drawing.
3. Many teachers used material furnished by insurance agents whose children were in their classes, or from such organizations as the Institute of Life Insurance, 60 East 42nd Street, New York City. This Institute, or its representative in other cities, lends without cost except for postage, the film, "Search for Security," which portrays some of the history of insurance. This same organization supplies, free of charge, copies of the "comic book," THE MAN WHO RUNS INTERFERENCE, which portrays some of the purposes and values of life insurance.
4. Bankers have come in to discuss the services that banks perform.
5. Owners of small businesses came in and talked about problems of business such as profit and loss, taxes in business, pay roll deductions.
6. A city engineer showed the use of the transit to a trigonometry class and took the class on a surveying trip.
7. A state inspector for weights and measures discussed measurement.
8. A Social Security representative talked to a consumer mathematics class.

Some of the values from using these community resources are:

1. Contact with the workaday world.
2. Students learn that there are many sources of information besides the teacher.
3. There are fewer questions, such as "Why do we have to study this stuff?"

IV. Are We Doing Our Share in Guiding Pupils into the Mathematics Courses Best Suited for Them?

The Guidance Pamphlet in Mathematics, written by the Postwar Planning Commission, is the best help along this line that we have had. It was published originally in the November, 1947 issue of The Mathematics Teacher and is available from the Editor, Dr. William D. Reeve, 525 West 120th Street, New York 27, N. Y. for 10¢ each if ordered in quantities of ten or more. Individual copies are 25¢. This pamphlet has been valuable to students, teachers, counselors, principals, parents. Student find it helpful in answering their questions about what mathematics preparation is needed for various jobs and professions; they also enjoy checking their knowledge against the check list of "essentials of functional competence," which it contains. One Parent-Teacher Association purchased a large number of the guidance pamphlets in order to do a better job at home of counseling their children.

V. Are We Enriching Our Mathematics Teaching with Historical Materials so that Students will Understand the Struggles that Mankind has had in Order to Give us this Useful System of Thinking that We Call Mathematics?

A quotation over the entrance to the University of Colorado Library says: "Who knows only his own generation remains always a child."

Ever since prehistoric days, mathematics developed as it was needed: the early shepherds devised ways of tallying their sheep to keep track of them; the early Egyptians developed mathematical formulas to survey their land after it was flooded by the Nile each year; they also developed fractions (though with only unit numerators—except $\frac{2}{3}$) to help in collecting taxes when people owed only part of a bushel of grain. Various mathematical units of length were developed in different countries as needed: the cubit mentioned so often in the Bible, the foot which was usually the length of some king's foot, the yard which was usually the length of some other king's arm, the *mille passum*, meaning 1000 paces which became our mile, the furlong which was the "furrow long," the length of the furrow in the usual acre strip of the medieval manorial system, the rod which at one time was designated as the total length of the right foot of the first sixteen men who came out of a church on Sunday morning.

Many historical materials are available for teacher and pupil use. I will mention only a few. The pamphlet, *How Long is a Rod?*, is available for the asking from the Ford Motor Company, Detroit. The movie, ORIGINS OF MATHEMATICS (World Films, Inc. 1250 Avenue of the

Americas, New York 20, N. Y.) gives a glimpse of the history of our number system. When studying geometric figures, some of our students have enjoyed copying the medieval masons' marks from Matila Ghyka's *Geometry and Life* (Sheed and Ward, New York) and designing similar ones of their own. The pamphlet, *Numbers and Numerals*, by Smith and Ginsburg, from Bureau of Publications, Teachers College, Columbia University, gives an interesting condensed history of our number system. *A Short History of Mathematics* by Vera Sanford (Houghton Mifflin) gives much valuable information about the history of all branches of mathematics.

VI. Are We Using the Many Enrichment Materials which Give Life and Interest to Abstract Mathematics?

Someone may say, "Well, equations are abstract algebra; why not face that fact?" But what interest some classes have found in that famous equation $e = mc^2$, the formula for atomic energy! At the October 1948 meeting of the Academy of World Economics in Washington, that formula was called one of the greatest examples of abstract thinking in history. How thrilled some of our students have been to learn that many of the facts about atomic energy were discovered first by Einstein, and, according to General Groves, who was in charge of the Manhattan Project, not until the world-shaking experiment in New Mexico were scientists sure that the mathematics was correct. One of our teachers arranged a bulletin board entitled "$e = mc^2$" and students brought pictures to show the scientific results of the mathematical discovery. It enlarged their vision of the part that mathematics has played in the development of civilization.

Sometimes I think of the study of mathematics as a long hike through the woods. For some, it is a long, wearisome journey, with eyes fixed on the trail, never looking about them. For others, who are fortunate enough to have guides who call their attention to interesting plants or birds, gorgeous vistas, or beautiful cloud formations, the trip is a delightful one. Where shall we find the interesting things in our mathematical journey?

First of all, there are books—textbooks as well as supplementary books. In our Washington courses of study in mathematics, we reserve one column alongside the topic being taught to list the materials and references which are particularly helpful and interesting in teaching that topic. I will mention only a few of the books we have found particularly interesting:

1. *Mathematics and the Imagination* by Kasner and Newman (Simon and Schuster) has much interesting material about topics taught in high school mathematics.
2. *Benchley Beside Himself* (Harpers) has a delightful chapter on Roman numerals entitled, "When not in Rome, why do as the Romans did?"
3. *Mathematical Recreations* by Kraitchik (Norton) has a wealth of number puzzles and problems.
4. *Romping Through Mathematics* by Raymond Andersen (Knopf) is being widely enjoyed, as evidenced by its interesting book reviews.

Many teachers find that a shelf of mathematics books in a classroom furnishes much interest in mathematics. In an appendix of the 15th yearbook of the National Council of Teachers of Mathematics, there is a fine list of mathematics books for the high school library. Having a shelf of interesting mathematical books in the classroom motivates the study of mathematics for some students and takes some care of some of the problems of individual differences.

The enrichment afforded by books is so obvious and has such unlimited possibilities that we had better turn to a few less common means of enrichment. The new section in The Mathematics Teacher entitled "Aids to Teaching" is extremely helpful. Many teachers say that it alone is worth the price of the magazine. For instance, in the May, 1948 issue alone,

are listed and reviewed the following:

1. Booklets: *Men of Science* from Westinghouse. Free
   Navigation from Air Age Research. Free
2. Chart: *Biggest and Littlest Things in the Universe.* Westinghouse.
3. Equipment: Draft kit.
4. Films (four are reviewed)
5. Filmstrips (16 plane geometry filmstrips are reviewed).
6. Models: Multimodel geometry construction set from Yoder Instrument Co.
7. Sources of materials for laboratory work.

Professor Stephen Corey, now at Teachers College, Columbia University, once said that a good way to find out what is being learned in a school is to take a look at the curriculum materials the teacher is using. "These materials," he says, "which give a good hint about the kinds of things the students are DOING, tell more about a curriculum than can be told by a printed statement of its objectives." In the Armed Forces, they said, "Audio-visual aids are no longer regarded as a fad; their effectiveness has been proved in the cauldron of battle action where 'know-how' means survival."

After the visit to many schools in the Armed Forces by a group of Michigan educators under the leadership of Dr. Raleigh Schorling, they wrote a report called SWORDS INTO PLOUGHSHARES (State Department of Education, Lansing, Michigan) in which they made six recommendations concerning what civilian education can learn from education in the Armed Forces. The first recommendation was this: *We need to make greater use of sense experiences in staging learning situations.* What sensory aids shall we use? The 18th yearbook of the National Council of Teachers of Mathematics entitled *Multisensory Aids in the Teaching of Mathematics* is a treasure. (Bureau of Publications, Teachers College, Columbia University). On the first page it says: "We know . . . that for *our* pupils, learning *must* be something more than seeing and hearing; for them mathematics *must* be a means for *doing* things in the classroom as well as in a later workaday world." This idea is not new. In 445 B. C. Sophocles said, "You must learn by doing the thing; else how will you know that you can do it unless you try?" This yearbook is a compilation of teaching aids used by hundreds of mathematics teachers and should be in every mathematics classroom in the country.

Concerning the value of visual materials the use of color in mathematics textbooks is discussed in the 18th yearbook. One of the first books on plane geometry to do that was entitled *The First Six Books of the Elements of Euclid in which Colored Diagrams and Symbols are Used Instead of Letters for the Greater Ease of Learners,* by Oliver Byrne in London, 1847. One principle set forth in it is a quotation from Horace:

> "A feebler impress through the ear is made
> Than what is by the faithful eye conveyed."

The 18th yearbook also discusses slides, films, film strips, and descriptions of how to make them. Many new films are being produced for mathematics classes. A few which we have found helpful and interesting are:

FILMS (16 MM.)

Bray Pictures Corporation, 729 Seventh Avenue, New York.
Mysteries of Snow (silent)
(Geometry of snow crystals)

Coronet Instructional Films, 65 E. South Water St. Chicago 1, Ill.
Language of Graphs
Fred Meets a Bank (color)
What is Money?
Measurement

Encyclopaedia Brittanica Films, 20 North Wacker Drive, Chicago 6, Ill.
Using the Bank
Property Taxation

Young America Films, 18 E. 41st St. New York 17, N.Y.
Meaning of Percentage

Castle Films, Inc., RCA Building, New York 20, N.Y.
Slide Rule

FILMSTRIPS

Distributed by Jam Handy Organization, 2821 E. Grand Blvd., Detroit 11, Michigan.
Plane Geometry (color)
8 films—Introduction to geometry
2 films—Postulates
1 film—Locus

1 film—Geometry and art
4 films—Logic in geometry

Others can be found in film catalogs and some will be reviewed from time to time in "Aids to Teaching" in THE MATHEMATICS TEACHER. Two commercial films which we have enjoyed using are:

Precisely so—from General Motors, Dept. of Public Relations, 1775 Broadway, New York City. This film gives a splendid pictorial history of measurement.

Time—from King Cole Sound Service, 340 Third Avenue, New York 10, N.Y. This film was made by the Elgin Watch Co. and pictures interesting phases of progress in the measurement of time.

VII. ARE WE ENCOURAGING OUR MATHEMATICS STUDENTS TO EXPRESS THEMSELVES IN AS MANY ORIGINAL AND ARTISTIC WAYS AS POSSIBLE?

Some such possibilities are in:

1. Poems
2. Assembly programs in mathematics
3. Radio programs
4. Television programs
5. Collecting quotations
6. Cartoons
7. Debates

Over and above all the things that have been mentioned, the spirit of our teaching is what counts. "He who would kindle another must himself glow" as someone has put it. People do not act upon being informed but upon being inspired. One mathematics teacher, Frances Johnson, has expressed this thought exceedingly well:

It is not the arithmetic of our prayers, how many they be,
Not the rhetoric, how eloquent they be,
Not the geometry, how long they be,
Not even the divinity, how exact the doctrine may be,
That God calls for, but the
Fervency of the spirit is that which availeth much."

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A Summary Report of the Second Annual Workshop for Teachers of Mathematics at Galesburg

By Daniel W. Snader and Ann Bromley
University of Illinois, Galesburg, Ill.

At the University of Illinois, Galesburg, Illinois, the Second Annual Workshop for Teachers of Mathematics was held last summer under the direction of Professor Daniel W. Snader. Problems of the curriculum, of testing and evaluating, of vitalizing instruction through the use of instructional aids and instruments, of the psychological approach to the teaching of arithmetic, algebra and geometry, and of the applications of mathematics to living and working were considered in a program consisting of general sessions and smaller study groups.

Professor Willard B. Spalding, Dean of the College of Education, University of Illinois, addressed the first general session on "Education Looks Ahead." Professor John R. Clark, Teacher's College, Columbia University, chose the interesting topic, "How the High-Grade Illiterate Does His Arithmetic," for his address at the Workshop Banquet.

The five study groups were scheduled for morning and afternoon sessions and were selected by the participants according to their professional interests and needs.

A Psychological Approach to the Teaching of Arithmetic

By Professor John R. Clark

This group studied various methods of meeting the inadequacies of arithmetic teaching and examined the implications of the new psychology on that teaching with regard to (1) the nature or concept of arithmetic, (2) the learning of concepts and generalizations, and (3) the thinking and reasoning involved.

The idea that the newer psychology emphasizes relationships rather than specific facts was discussed. For effective learning the *pupil* should discover these relationships himself from the consideration of many particular situations. An objective in the teaching of arithmetic should be to help the pupil to acquire the ability to analyze the problem situation in terms of the number system. The problem of learning concepts and generalizations can be attacked by *teaching* for transfer from the concrete to the abstract. To be meaningful an arithmetical concept must be fitted into the number system and in the evaluation of arithmetical learning, the concepts of arithmetic are being weighted more and more heavily. A purpose of the study of arithmetic is to enable the student to handle the quantative thinking with which he is confronted. Arithmetic should be so taught that the pupil can work his way through new situations involving numbers. Teachers must teach for problem analysis as well as for answers, in fact even more so.

A Psychological Approach to the Teaching of Algebra

By Professor John R. Clark

An analysis of the types of learning in algebra with regard to (1) algebra as generalized arithmetic, (2) functionality as the basic concept of algebra, (3) the role of algebraic skills, concepts, and problem solving in algebra, (4) the teaching procedures best designed to achieve algebraic learning and (5) the learning difficulties in algebra were all considered.

Teachers of algebra should correlate the study of algebra and arithmetic and emphasis should be placed upon the *discovery* of the relationships between algebra and science, business, agriculture, and technology. Problem solving requires algebraic meanings and skills, and those skills and concepts needed in problem

solving should be stressed so that meaningful learning rather than the mechanics involved can be acquired by the pupil. In developing power in problem solving, there should be practice in *oral analysis* and pupils should be encouraged to "guess, and check the guess." Problem solving should result in great improvement in the reading comprehension of many pupils. Prevention of learning difficulties can be brought about by basic understanding and meaningful practice rather than by repetitive drill.

Vitalized Instruction in Mathematics Through the Use of Instruments

By Professor Daniel W. Snader

Teachers of junior and senior high school mathematics are becoming increasingly aware of the contributions which the use of instruments such as the slide rule, hypsometer-clinometer, angle mirror, sextant, transit, plane table and alidade, etc., can make to the regular class room instruction.

The members of Study Group III studied:

1. The mathematics involved in the construction of each instrument named above.
2. The many different uses to which each one can be put in teaching junior and senior high school mathematics.
3. The techniques for teaching and applying the fundamental principles of each instrument.
4. Special projects in correlating shop work, art, science and mathematics.
5. The problem of developing a reserve of source materials for instructional purposes.
6. The organization and preliminary preparation for special field projects in practical mathematics.
7. The availability and cost of the necessary mathematical equipment for enriching and vitalizing the entire mathematics program.

Special emphasis was placed upon the use of instruments in teaching decimals, ratio, proportion, similarity, congruence, indirect measurement, meaning of angles of elevation and depression, trigonometric functions, approximate computations, mapping (scale drawings), and simple surveying projects.

Slide rule instruction was shown to be appropriate for seventh grade instruction. When used properly, it is an effective vehicle by which to review decimals and develop real number sense.

Instructional aids in the Teaching of Mathematics

By Martha Hildebrandt, Head of Mathematics Department Proviso Township High School, Maywood, Illinois

A great many pupils studying geometry and trigonometry experience considerable difficulty with these subjects because of their inability to visualize the relations being studied and the relationship existing between geometric facts and life situations. It was the aim of this group to study aids in teaching according to: (1) those made of easily available materials—paper, tacks, rubber bands, string, etc., (2) those aids which require more time and energy in planning and construction—materials made at home or in the school shop by either pupil or teacher, (3) commercial aids and their use, and (4) pictures, moving pictures and film strips.

An extension of the use of these aids by teachers can improve the understanding of the subject by the students and increase the popularity of the subject.

Seminar: Problems in the Teaching of High School Geometry

Each day a different topic related to the problems of teaching high school geometry was considered in this seminar. Professor Gertrude Hendrix of the University High School, Eastern Illinois State College, Charleston, Illinois led the group in a discussion of "The Analytic Method of Teaching Geometry." Illustrations were made to the actual teaching of

geometric proof, pointing out that at least one generalization is the necessary mental equipment for each step and that it is unnatural to pursue the use of either the analytic or synthetic method exclusively. A practical method for eliminating unnecessary steps in a formal proof was presented; by means of this device, the main steps of the argument will stand out in bold relief and purely formal particulars will be made subordinate.

"The Fundamentals of Critical Thinking in Geometric and Non-Geometric Situations" were discussed by the group under the leadership of Professor Hendrix. The distinction between explanation and prediction must be clarified before suggestions for positive and systematic thinking about any problem can be made—(1) recognize and formulate the problem, (2) collect and organize the data, (3) interpret and state the conclusion, and (4) carefully evaluate and test. One of the greatest factors in critical thinking is to develop a sensitiveness to inconsistency. The technique for realizing inconsistency in an argument can be developed in the pupils by the teachers of mathematics and particularly by the teachers of geometry.

Mr. John F. Schacht of the Bexley High School, Columbus, Ohio, conducted the seminar on "The Use of Instruments in Vitalizing the Teaching of Geometry." The use of models will (1) make the study of geometry more interesting, (2) contribute to the achievement of one of the principal purposes of teaching geometry, namely, the improvement of the ability to think clearly, (3) help visualize geometry for the slow learner, (4) encourage the habit of attacking problems by looking for relationships, and (5) provide additional drill for the geometric propositions and original exercises. The experimental-laboratory procedures employed in using models are a frequent source of problems proposed by the students themselves, and often times these problems have more educational potential than those assigned from a text.

Testing generally asks "How much" while evaluation insists upon knowing "Of what value." The distinguishing factors between these two were discussed during the seminar session on "The Testing and Evaluation Program for High School Geometry" under the leadership of Professor Bjarne R. Ullsvik of Illinois State Normal University at Normal, Illinois. A good evaluation program must (1) be comprehensive and continuous, (2) be diagnostic and remedial in character, (3) provide curricular freedom, (4) consist of certain technical qualities, (5) be capable of intelligent interpretation, and (6) allow for pupil-teacher participation. The following objectives for a course in plane geometry were considered: To develop (1) the ability to think logically, (2) the art of deductive thinking, (3) the correct patterns of expression—written, spoken and symbolic, (4) the concept of proof or verification, (5) the elemental or sensory type of proof, (6) an appreciation of beauty of form, and (7) the practical, cultural, and vocational uses of geometry.

Professor H. G. Ayre of Western Illinois State College, Macomb, Ill. focused the attention of the group toward the "Curriculum Problems Related to High School Geometry." The voluminous writing on curriculum problems might tend to make us believe that revision of the curriculum would serve as a panacea for all of our problems; however, regardless of all that has been written on the subject, we still have curriculum problems.

There should be a two-track program in geometry which would provide sound training for the students who have talent in mathematics and sciences and also insure functional competence for the everyday affairs of life to develop a more intelligent citizenry. Such problems as: (1) How many theorems should be required in geometry? (2) Should non-mathematical material be included? (3) Should all pupils be required to take geometry? and others were considered and discussed by the group.

Mathematics Used in Industry

In order to fuse more closely the relationship between business, industry and mathematics, various representatives from industrial concerns discussed the mathematics used in their particular industries. Mr. B. L. Hummel represented the Hamilton Watch Company of Lancaster, Pennsylvania, while "A Mathematical Analysis of Mechanisms Used in Automatic Machines" was discussed by John W. May, Chief Engineer of Design and Research, of the Wright Automatic Machinery Company in Durham, North Carolina. An illustrated lecture combined with audience participation on the Monroe Calculating Machine was given by Mr. E. M. Spoerer, the Chicago representative of this company; special emphasis was placed on the value of these automatic machines in teaching mathematics. "Some Applications of Mathematics to the Problems of the Tire Industry" was the topic R. D. Evans of the Goodyear Tire and Rubber Company, Akron, Ohio, presented to the group. In this manner, theory and practice, mathematics and its applications in industry were tied together and the definite transfer between the two were more concretely viewed. Teachers of mathematics, especially general mathematics of grades seven, eight and nine, have found these lectures very helpful as source materials for enriching and making more meaningful and effective their teaching of general mathematics.

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Fun with Mathematics

An Assembly Program

Arranged by Irving Adler, Chairman, Department of Mathematics
Straubenmuller Textile High School, New York 11, N. Y.

Cast: One boy who serves as M.C., and nine other pupils, including at least one girl.

M.C.: When you wrestle with numbers in a mathematics class, you sometimes wonder who is the boss, you or the numbers. You work and ponder, and squirm and finnagle, and still the numbers refuse to behave! The way they keep giving you the wrong answer it looks as though they purposely set out to trip you up. But I'll let you in on a secret. That only happens with numbers that haven't been tamed yet. If a number has been broken in properly you can really get it to do anything you like. Let me show you. (Turns to pupils at board.) Everybody write the number 12345679. (To first pupil) Do you have a favorite number?

1st P: Yes. My favorite number is 4. (Writes it on board.)

M.C.: Then multiply by 36. (Proceeds down the line.) What's your favorite number?

2nd P: 7.

M.C.: Multiply by 63. And yours?

3rd P: 2.

M.C.: Multiply by 18. And yours?

4th P: 8.

M.C.: Multiply by 72. And yours?

5th P: 3.

M.C.: Multiply by 27. And yours?

6th P: Mine is 1.

M.C.: Multiply by 9.

7th P: Mine is 9.

M.C.: Multiply by 81.

8th P: Mine is 5.

M.C.: Multiply by 45.

9th P: Mine is 6.

M.C.: Multiply by 54.

M.C.: (Returns to first pupil) What was your favorite number?

1st P: 4.

M.C.: What answer did you get?

1st P: 4! 4! 4! etc.

(M.C. proceeds down the line. Each pupil in turn reads his favorite number and then he very emphatically reads his result.)

M.C.: If you tame your numbers properly you can do many remarkable things with them. You can even do mind-reading. Watch this! Blindfolds, please! (M.C. is blindfolded).

M.C.: Now, everybody—write any three-digit number. Reverse the digits and subtract. Now multiply by any number you wish. Cross out any digit except 0. Add the remaining digits. Now give me your results.

1st P: My result is 11.

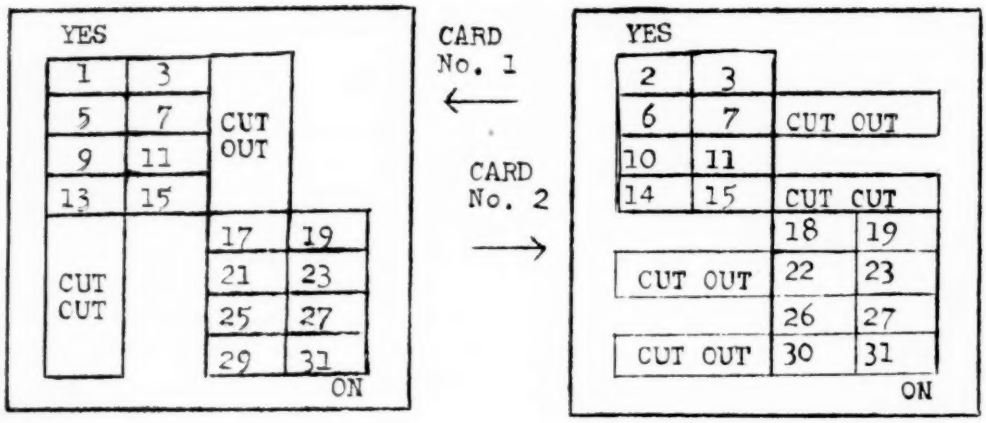
M.C.: You crossed out a 7.

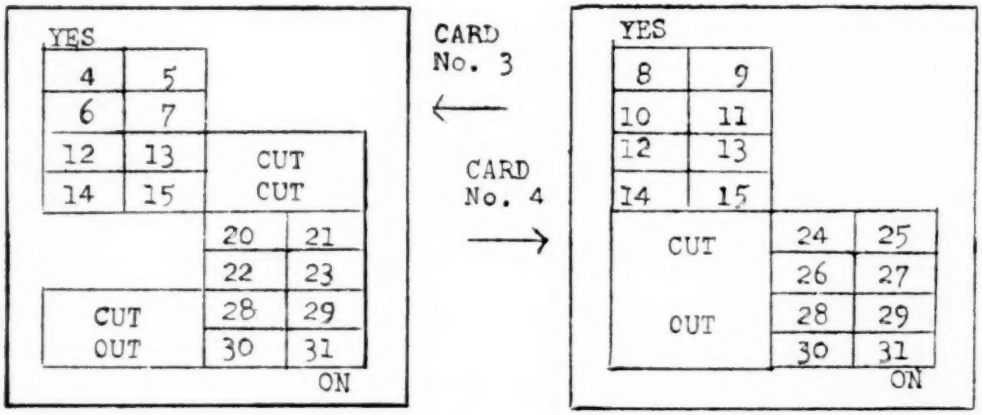
1st P: That's right! (This is continued down the line. The M.C. "guesses" the number crossed out by adding the digits of the number read to him, and subtracting from 9.)

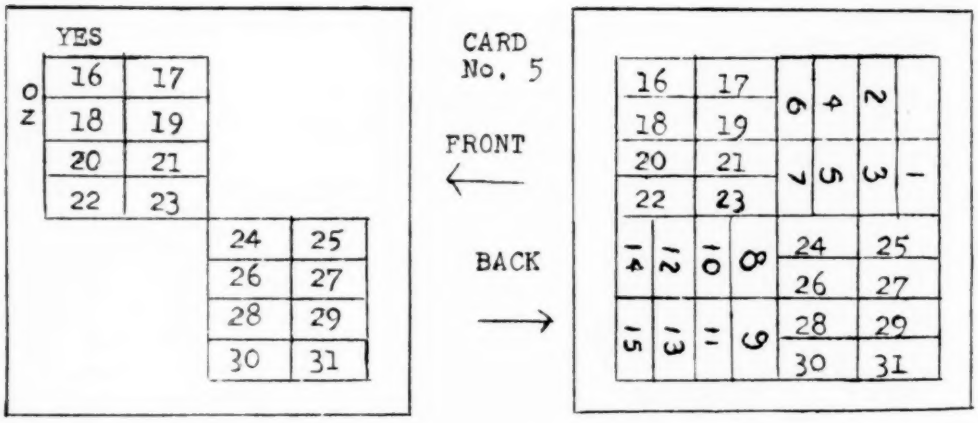
M.C.: The next trick requires the use of paper. Will someone from the audience volunteer to assist me with this one?

(Two pupils hold a large sheet of paper, while the volunteer writes on it with crayon.)

M.C.: Write any three-digit number in which the first and last digits differ by more than one. Now reverse the digits and subtract. Reverse again and add. Show your answer to the audience. Now fold the paper several times and give it to me. (Removes blindfold.) The answer is on the paper. (Lights a match and burns the paper.) Now the answer is in the ashes! Now I shall get the answer from the ashes! (Picks up the ash, and rubs it on his arm. The answer, 1089, previously written in milk, shows up in black and is shown to







Directions for making the age-guessing cards: Use cards large enough to be read from the audience. Copy these models.

the audience.) Here is the answer!

M.C.: One of the biggest mysteries in numbers is a woman's age. (Turns to girl) How old are you?

Girl: I won't tell. Try and find out!

M.C.: I will! Write your age on the board so the audience can see it and then erase it. (Girl does so.) Bring out the magic cards. (Cards are brought out. Directions for making the cards are appended to this text.) Is your age on this card? (This question is repeated with each card. Girl answers yes or no. Cards are placed against board with yes or no on top, depending on the girl's answer. After last card is put down, M.C. picks up whole set and turns it around to show audience the girl's age, visible through the window.) Here is your age!

M.C.: Thank you very much for helping me. Now if you'll just step off the stage, I have one more trick that I can do alone.

All pupils: No! We want to stay! Nothing doing! No sir! (General uproar.)

M.C.: Now there's no use shouting. You have to get off!

Girl: (Edges up to M.C. Seductively.) You don't want me to leave, do you?

M.C.: Yes, everybody. Well now, that is, on second thought, you can stay. But everybody else, off!

All: No sir! If she stays, we stay! We won't get off. Try and make us! etc. (All crowd around M.C. menacingly.)

M.C.: All right! All right! Quiet! Let's do it the fair way. Everybody form a circle. (To girl) You stand here. We'll count around the circle, and every third person goes off. You begin. (They count off, 1-2-3, 1-2-3, etc., until only M.C. and the girl remain. Directions for arranging the circle are appended to this text.) (To audience) You see, if you know your numbers you can't lose! (To girl) Ready? Write a three-digit number whose first and last digits differ by more than one. Reverse and subtract. Now reverse and add. Multiply by one million. Subtract 965, 543, 282.

Under every 1 write O.
Under every 2 write G.
Under every 3 write E.
Under every 4 write H.
Under every 5 write T.
Under every 6 write N.
Under every 7 write I.
Under every 8 write J. Now read it backwards.

(Pupils backstage stick their heads out from behind blackboards).

All: (Shout in unison) JOIN THE G. O.! (Curtain.)

Directions for arranging the circle: If the positions in the circle are numbered from 1 to 10, in the direction in which the counting takes place, let the M.C. take position 10, the girl position 4, and let the person in position 1 start the counting.

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How Can We Improve Instruction and Achievement in Arithmetic?

By Dolphus Williams
Sanger Union High School, Sanger, California

To people who are not actually working with mathematics, but who require calculations in their affairs it seems, all too often, that the helpers they employ are short in their skill at handling figures. The criticism, also, by the Armed Forces that the draftees lacked mathematical skill requires some explanation.

First of all you can not tell much about what one really knows by requiring him to underline this number and circle that one, etc. We do not solve our mathematical problems in life in that way.

Secondly, the boys were not in a condition emotionally to do their best under the circumstances. And thirdly, most of them had been removed from studying mathematics by at least two or more years. Psychologists, who claim to know, say that we forget within two years at least seventy per cent of the facts we learn. Consequently at the time the boys were drafted, since many of them had probably not worked with mathematics after the sophomore year in high school, the chances are that they had already forgotten one or more of the processes that were required in most of the problems, even though they had once known it. Consequently, since the answer in mathematics is definitely either right or wrong, the score would necessarily be low.

I believe that it is this measuring stick of accuracy required in mathematics, which lowers the score for the student of mathematics, rather than any greater lack of knowledge in mathematics compared with knowledge in any other subject.

Moreover, when we admit, as we must if we think this matter through, that mathematics is the thread required to bind together the knowledge of astronomy, the physical sciences, electrical and chemical sciences, as well as industry and business, we shall not be too ashamed of our accomplishments in mathematics. When we realize the marvelous advances made in all of those lines which require mathematics, and compare them with the solution of our political, religious, and social problems, are we so far behind?

Let me illustrate this difference between the subject of mathematics and a less exact subject, such as English for instance. A couple of days ago, I asked one of the boys in the advanced algebra class what was meant by "Consecutive Integers." He replied by saying, "Integers that follow one another in uninterrupted succession." I asked him to illustrate. He said χ, χ square, χ cube etc. He should of course, have said χ, χ plus one, χ plus two, etc.

In an English class his definition would have been 100% correct. But his understanding of its meaning in algebra was completely incapable of securing the right answer and so would have been 100% wrong. In mathematics, we must not only say the right words, but we must understand them. There is no middle ground. The answer is either right or wrong. In other subjects where answers are less definite we may be only on the verge of understanding, and still say the right thing, and thus appear to know more than we do.

As to their getting better and quicker results in the army in their teaching of mathematics, "it is to laugh." Give the grade teacher only a half dozen pupils, and let her work with them all day long on one process, for weeks at a time, as they did in the small instruction groups in the army, and at the same time lead the pupil to think that he will be shot if his attention to her wavers for a single moment, I as-

sure you the teacher will also get results and efficiency in arithmetic more quickly in our public schools. But along with good results and efficiency in arithmetic, we would also fill up our mental institutions, as the army hospitals are already learning. No one but the army would desire such proficiency at that price. However, we have no objection to using more efficient methods gleaned from other fields for we are always seeking better ways to get our instruction across. May I offer the following suggestions that, in my experience, have proved of value for learning mathematics.

What should be the objectives of the teaching of arithmetic in the eighth grade?

The first objective should be to have no more than twenty pupils in each arithmetic class. With no more than twenty pupils in each class the teacher could keep all of the pupils at the black board a major part of the time, drilling them on the fundamentals. What I mean by the fundamentals is addition, subtraction, multiplication, and division of whole numbers and fractions. This would include decimal and common fractions, changing from common to decimal, and from decimal to common fractions and, of course, much drill on the multiplication tables.

I am confident that if each pupil averaged only fifteen minutes each day at the blackboard drilling on fractions and whole numbers, with no thought as yet of their application, they would be far better prepared for high school work than they are today. Because when they came to their algebra problems they would then know how to do the various fundamental processes that are required to solve the problems. It does little good to know what has to be done if we do not know how to do it. It is at just this point that interest flags.

Teachers should make the learning of fundamentals in arithmetic as pleasurable as, for example, is done in music and athletics. If the pupils all work the same problems together, it will be no more boring, monotonous, or uninteresting to them, than is band practice, orchestra, basket ball, or base ball practice etc., except that these have the added incentive of public performance.

If, quite frequently, in all the grammar schools throughout the county, on a Friday afternoon for an hour and a half, time would be given to spelling and ciphering matches, as of old, we should see improvement in skill at handling the fundamentals of arithmetic. There should also be an occasional competition at night to which parents would be invited. Then, if, in addition, the county office would sponsor a spring county contest for the best pupils in spelling and arithmetic, we should see more enthusiasm for arithmetic and spelling, not only in the students but in the parents and teachers as well.

Make learning of arithmetic a job in itself with no thought of its ultimate application and the pupils will respond more willingly. Then, after they have mastered the fundamentals, train or test them on the practical applications.

Children, or grown ups for that matter, are not much interested in rewards too far off in the future. When we begin to talk about the future use of mathematics we start their minds to believing it is work, and that is something that we all like to avoid.

With the upper half of almost any class, I should expect these pupils to apply their skills on the verbal problems, as a reward for good work, and after they had become proficient in the manipulation of fractions.

It is my belief that it is a waste of time to try to get the lower half of most classes to understand the working of the verbal problems. They should be required, at least, to master the handling of fractions before any time should be spent with them on the verbal problems.

A major part of the drill work should be done at the blackboard because it will be easier to keep the pupils busy if they are standing. In general, they like it better.

They can learn from others while catching on, and in developing speed and accuracy, can check themselves with others. At the same time, it will take less energy for the teacher to check twenty pupils at one time at the blackboard, than it will to check one paper.

Pupils enjoy developing speed and getting a lot of answers right. I have had some of the toughest fellows in school, whom I had forced to stay at the board until they caught on, exclaim as they left the room at the end of the period: "Oh that was fun." We all enjoy doing a thing if we can do it well.

If the pupils are assigned a number of problems to do at their seats, but do not understand clearly how to tackle them, the chances are that they will not try to do them and will likely waste their time making aimless drawings or in creating a disturbance, while at the board they will at least, be working.

I think we are wasting our time and energy if we try to find practical applications in order to stimulate interest. Children are not interested as yet in the applications but rather just in getting the lesson for the day. However they are interested in, and thrilled at doing things well, and in excelling.

As to the verbal problems, strange as it may seem, pupils are not so much interested in finding out how much sand, gravel, or cement is needed in making a foundation for a building as they are in finding the age of Mary if she lacks two years of being twice as old as James.

Mastery and recognition is enough incentive to the child in learning any subject assigned. We should continually drive home to the youngster that his job is to learn facts and to develop skills. He should understand that his job of learning is just as much of a business as it is his mother's business to keep house or his father's to supply the funds to finance the home.

I am not worrying about the fact that the child in the lower group is not learning to work the verbal problems. If he retains the knowledge of how to handle fractions he will have little trouble in working the everyday problems that he will meet when he becomes an adult. As for the upper group, they will have been initiated into the verbal problems and will go on to master them in the high school.

The time to learn these skills is early in life, a time when it is just as much fun to learn to add $\frac{2}{3}$ and $\frac{7}{8}$ as it is to learn how to ride a bicycle. Neither is much fun when one becomes an adult but both are thrilling at the proper age, which is in childhood.

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AIDS TO TEACHING

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This article completes a year's series on materials of instruction available for mathematics teachers. We hope to continue the series. The interest evidenced by the comments and suggestions that many of you have made has been appreciated. We have tried to give accurate descriptions and appraisals of many types of teaching aids. However, we would appreciate your participation in the critical evaluation of materials. We want you to share your experiences with teaching materials with other mathematics teachers. Send us information, experiences, ideas and suggestions about teaching aids to be published in this department under your name, so that this series can serve as a clearing house through which mathematics teachers can exchange information about teaching materials.

BOOKLETS

B. 19—*Pursuit of Accuracy*

Hamilton Watch Co.; Lancaster, Pennsylvania.

Booklet; 28 pages; free; 1940.

Description: This booklet presents a short review of mankind's persistent effort to devise more and more accurate means of measuring time from the first crude sundial to the invention of the mechanical clock. It discusses briefly the place of the sun, stars, fire, and water in early time measurement. Then it describes the mechanical development of the modern watch. A world map shows differences in time in different cities. The last half of the booklet is devoted to advertising the Hamilton watch by describing its manufacture.

Appraisal: This booklet will furnish supplementary material for a unit on time. However, it is not written in a style to appeal to junior high school pupils. It contains a considerable amount of advertising.

B. 20—*Taxes*

Building America; 2 West 45th street, New York.

Booklet; 32 pages; 30¢; 1942.

Description: This issue of *Building America* discusses taxation and illustrates the major points with many photographs and charts. The following questions are discussed: What kinds of taxes do we Americans pay and what do we get in return for our money? What is the history of taxes in our country? What are the principal systems of taxes in America and what kinds of taxes do these systems levy? Are taxes levied on a fair basis? Are taxes too high? What can America do to improve its method of taxation? Several graphs are used to illustrate points. A bibliography of books, pamphlets and articles is included.

Appraisal: During the present period of inflation and high taxes, every citizen should be informed about the problems related to taxation. This publication discusses taxation questions in an interesting manner using many real photographs and graphs. The text is suitable for both junior and senior high school students. Although some of the material is out of date, it will be a valuable supplement for a unit on

taxation, particularly for those teachers who place emphasis on the social phase of mathematics instruction.

FILMS

F. 31—*We Discover Fractions*

Coronet Films; Coronet Building, Chicago, Illinois.

16 mm sound film; 1 reel; Black and white—$45, color—$90; 1948

Content: A boy and his father cut boards into two and four parts when building a bookcase. These parts of boards are used to illustrate the meaning of one-half and one-fourth. A ruler divided into twelve parts pictures the fractions 1/12 and 2/12 and the addition of these fractions. A girl and her mother use fractions in baking a cake according to a recipe. Similarly a piece of pie, an apple and a box of pencils present fractions in familiar terms that clarify the meaning of numerator and denominator. The summary is presented in a unique manner showing the facts about fractions which a pupil is writing in his notebook.

Appraisal: This film should improve pupils' understanding of fractions since the terms are related to concrete objects. However, the film should not supplant the use of concrete materials by the teacher. It will be useful in showing ways to use concrete material to make abstract principles meaningful. The film combines pictures of real situations and animation in a manner that should hold the attention of pupils. The narration and sequence of scenes has good continuity. It should be useful to introduce the meaning of fractions or as a review of the meaning of fractions in grades 3 to 5. The photography and commentary are satisfactory.

F. 32—*Installment Buying*

Coronet Films; Coronet Building, Chicago, Illinois.

16 mm sound film; 1 reel; black and white—$45, color—$90; 1948.

Content: A young doctor, who has just gone into private practice, needs furniture for his waiting room but cannot afford to pay cash for it. His experience in buying furniture on installments and conversation with patients about their experiences with installment buying illustrate the pitfalls of installment buying. The film encourages the viewer to make a complete investigation of installment credit, contracts and interest rates before making a purchase in this way. Three questions to answer before buying on credit are emphasized: (1) Is the article worth buying on installment? (2) Can you afford it? (3) Are you getting the best installment terms?

Appraisal: Although this film was produced primarily for business and economics classes, it has material that can well be used to supplement consumer mathematics instruction. Although no problems are worked in the film, the figures quoted will furnish a good basis for problems on installment charges and rates. The difference in given rate and true rate is mentioned but no method of computing the different rates is explained. It is fortunate that all the purchases mentioned in the film are those for adult needs. The purchase of a bicycle or sporting goods would have more appeal to adolescents. This film should not be used as a substitute for field trips by students to obtain information about installment purchases in the local community. The fact that the film is built about the purchases being made by a doctor may present installment buying as a more desirable practice than is justified. The photography is good and the commentary satisfactory but lacks interest for junior high school students.

FILMSTRIPS

FS. 43—*Introduction to Algebra*

Society for Visual Education, Inc.; 100 East Ohio Street, Chicago, Illinois.

35 mm filmstrip; 47 frames; Black and white; $2; 1948.

Content: This filmstrip shows different types of signs and symbols that are used

by man, what signs and symbols are used in algebra, and how to use signs and symbols in solving problems. Many types of signs and symbols are pictured, such as Egyptian hieroglyphics, alphabets, traffic signs, punctuation marks, and shorthand. Algebra is the language of signs and symbols using the symbols of arithmetic and literal numbers. The literal factors of formula are introduced as abbreviations for units of measure or unknown quantities.

Appraisal: This filmstrip will furnish the algebra teacher with interesting background material to use as an introduction to an elementary algebra course. It is particularly usable as an introduction to literal numbers. The translation of a statement into a formula in which every word is translated into a mathematical symbol is effective. However, the filmstrip is inadequate in showing the importance of algebra in everyday life. Instead, algebra and arithmetic are shown as a stairway to high mathematics. Several frames of questions and answers at the end waste film. The drawings are good and the titles adequate.

FS. 44—*Introduction to Equations*

Society for Visual Education, Inc.; 100 East Ohio Street, Chicago, Illinois.
35 mm filmstrip; 60 frames; black and white; $2; 1948

Content: This filmstrip purports to show what an equation is, where equations are used, and how to solve equations. It shows that an equation is an expression of equality whether it be an equality of numbers, equality of energy in foods, or equality of weights on a scale. A balanced scale is used to illustrate the terms of an equation and the solving of equations by addition, subtraction, multiplication or division. The axioms of equality are stated as the golden rule of solving equations. The filmstrip ends by presenting several questions and answers to these questions.

Appraisal: Although this filmstrip says that equations are used in business, industry and the home, it fails to show any real applications of equations. It would seem to the reviewer that the textbook and teacher can show what an equation is and methods of solving equations and that visual aids should furnish illustrations of their application. This filmstrip contains no photographs of actual situations, all frames being drawings and titles. It wastes some valuable frames in giving the answers to simple questions about equations. Not only are the answers unnecessary but they also do not follow the questions in consecutive frames. The drawings are fair and adequately labeled. It would be most appropriate to use this filmstrip as an introduction to the study of equations.

FS. 45—*Introduction to Circles*
FS. 46—*Common Tangents and Tangent Circles*
FS. 47—*Loci*

Society for Visual Education, Inc.; 100 East Ohio Street, Chicago, Illinois.
35 mm filmstrip; Black and white; $2 each; 1948

Content of FS. 45: This filmstrip of 50 frames shows the parts of a circle and how they are related to everyday things such as cars, machines, flowers, building designs, bridges, and belts on pulleys. The terms defined and illustrated are circle, circumference, diameter, chord, arc, semicircle, and tangent. A series of questions for the pupils to answer is given at the end.

Content of FS. 46: This filmstrip of 45 frames defines tangents, point of tangency, common tangents, and tangent circles. For each definition examples are given such as train wheels tangent to the rails, light rays tangent to the moon during an eclipse of the sun, belts tangent to pulleys and gears tangent to gears.

Content of FS. 47: This filmstrip of 52 frames defines a locus and states the five fundamental loci. Following the statement of a locus, an everyday example is pictured. The examples include airplane propellor,

highway lane, corner bisector, guard rails on a bridge, a football kicked midway between goalposts. Besides the five common loci of plane geometry, this filmstrip shows the parabola and ellipse as loci with examples such as the path of a football pass, searchlight reflectors and the orbit of the earth's revolution.

Appraisal of FS. 45, FS. 46 and FS. 47: These filmstrips will furnish the geometry teacher with good illustrations of common geometric terms. However, they should not be used as a substitute for concrete objects. The drawings are good and the titles adequate. However, they lack material that is closely related to adolescent interests. For example, the belt of a motor scooter would have been a more apt illustration of tangents than the pulleys and belt on a factory machine. Considerable film is wasted on questions and answers at the end of each strip.

INSTRUMENTS

I. 9—*Hypsometer*

Yoder Instruments; 140 North Market Street, East Palestine, Ohio.

Field instrument—$6; Jacobs staff—$4.

Description: This hypsometer combined with clinometer, slope table and graphic sine, cosine and tangent table is an instrument with which one quickly finds the height of an object, for example a building, without computation or use of mathematical formula. At the same time it is possible to read directly the angle of inclination in degrees, the percentage of slope and the sine, cosine, and tangent of the angle correct to three decimal places. In addition, this instrument may be used for a rapid leveling job in which case the slope of elevation or depression as well as the percentage of slope may be obtained. It may also be used to find latitude or the true north-south line.

Appraisal: This is an inexpensive article of equipment which should be used in every mathematics classroom where indirect measurement is taught. It is a device to be used in field work and classroom instruction in intuitive geometry, demonstrative geometry, and trigonometry in junior and senior high schools. In addition to its use as a field instrument, it can be used to demonstrate the graphic relations between various trigonometric functions. It is well made and simple to operate. A Jacobs staff should also be purchased for mounting the hypsometer for field work.

RESOURCE UNITS

R. 1—*A Tentative Resource Unit on Insurance*

Dr. Kenneth B. Henderson et. al.; College of Education, University of Illinois; Urbana, Illinois.

Resource Unit; Mimeographed; 81 pages; 50¢; 1948.

Description: This resource unit is a report of a curriculum workshop in mathematics at the University of Illinois. It provides complete suggestions on objectives, subject matter, learning experiences, evaluative procedures, and references for the teacher and students. Initial activities are suggested to introduce the subject, arouse interest, establish goals, and develop the desire to learn. These activities are suggested as a basis for pupil-teacher planning. The suggested learning experiences include a variety of activities such as the formation of a pupil insurance company, survey of home, school and community needs, collection and analysis of data, problem solving, and oral and written discussions. The unit covers all common types of insurance and social security. In addition, a section is devoted to insurance as a vocation. An extensive glossary of terms and bibliography of books, pamphlets, articles, and films are included.

Appraisal: This unit will furnish the progressive mathematics teacher with material for vitalizing her teaching of one unit and suggest a pattern for planning other units. It is well organized and comprehensive in its treatment. It contains a great number of learning activities and

references. It includes more subject matter and more activities than one would use in one class, but this makes it highly adaptable to differences in aptitude, interests, and abilities of students and teacher. This unit shows the classroom teacher a concrete example of educational theories such as pupil-teacher planning, evaluation in terms of objectives, learning through experience, the social phase of mathematics, adapting instruction to individual aptitude and interests, and using community resources.

TESTS

T. 2—*Larson-Greene Unit Tests in First-Year Algebra*

Robert E. Larson and Harry A. Greene; Bureau of Educational Research and Service; State University of Iowa; Iowa City, Iowa.

Two forms; six tests; each test approximately 40 minutes; sample set—45¢; per 25—$3.25; answer sheets—$3.25 per 25; 1947.

Description: This 24 page test booklet contains six tests covering fourteen units of the subject matter commonly taught in the course in first-year algebra from literal notation through quadratic equations. All test items are of the multiple choice type with answer sheets and scoring stencils available. Each item is a short statement of a specific problem with four possible answers given for each item. A fifth choice for "answer not given" is provided on the answer sheet. Scratch paper is used by the students for computational work.

The reliability coefficients for the six tests vary from .821 to .886. No coefficients of validity are given. According to the publishers, "The validity of these tests has been determined both statistically and in terms of classroom practices. Although the selection of content was determined largely by analyzing numerous modern textbooks and other professional literature, additional information on the sequence of units was obtained by circulating inquiries to a number of qualified mathematics teachers." The difficulty level and discriminating power of each item was determined experimentally by an analysis of responses given by 1900 students in 28 different high schools. Percentile norms for the total scores represent achievement in normal classes when tested immediately following the completion of instruction on the particular unit covered by the test.

Appraisal: This test undoubtedly satisfies technical criteria such as reliability, validity, and objectivity. However, it does not adequately measure all the objectives of a modern algebra course. For example, it does not measure adequately applicational learning since it lacks problems that illustrate real practical applications of algebra. It does not measure adequately the student's understanding of processes of solving problems since most responses are the final answers to problems. As all problems must be complete statements, it places emphasis on the student's ability to read problems in a form which is frequently quite different from the method of presenting problems in a textbook. Thus, the tests discriminate between students who have a thorough understanding of subject matter and those students who possess only the ability to recall formal rules, procedures, and definitions. Because the tests parallel the usual algebra textbook organization, they will be useful as unit achievement tests in the typical algebra classroom. Since the student does no work on the test booklets but enters all of his responses on separate answer sheets, the test booklets may be used repeatedly by different individuals and classes.

Make your reservations *now* for the Annual Meeting of The National Council of Teachers of Mathematics at the *Lord Baltimore Hotel in Baltimore, Md.*, March 30, 31 and April 1 and 2, 1949.

◆ THE ART OF TEACHING ◆

The Slide Rule in the Junior High School

By JOHN E. MACCUBBIN
Carroll Manor School, Baldwin, Md.

ONE OF the most reliable weapons in the battle against careless errors on students' mathematics papers is the practice of having the student ask himself, after solving a problem, "Is my answer reasonable?" However, this presupposes that the student has had adequate and continuous exercise in estimating and judging answers. Among the ways in which this important faculty can be developed and maintained is through the use of the slide rule.

At first sight, it might seem premature to introduce the slide rule below the senior high level; but it seems to me that the feasability of such a step depends upon our objective in introducing it. I have been using the slide rule with 7th and 8th grade students to good effect. In the beginning, it was used solely as an enrichment for the better students. Later, I found it helpful with all but the slowest—and I'm not too sure than even these latter have not benefitted to some extent: it is still too early to be certain.

Two problems, neither insurmountable, may stand in the way of a general acceptance of the idea of using slide rules in the junior high: 1) the availability of cheap, reasonably accurate instruments, and 2) the availability of junior high teachers trained in the use of the slide rule.

Even in these days of inflation, however, 10-inch slide rules accurate enough for 7th and 8th grade work can be purchased for about twenty-five cents. True, they do not always have the S, L, and T Scales; but how many junior high courses of study would call for the use of these scales? All of them have A, B, C, and D scales, and most have the C1 and K scales as well.

As for teachers, it seems to me that teacher-training institutions might well give serious consideration to this idea. Even if the prospective teacher may not expect to teach mathematics, an elementary knowledge of the slide rule saves untold time in figuring averages and percentages for marked papers, attendance, and the many clerical chores with which most teachers are saddled. Besides, there seems to be a definite trend toward the use of the slide rule and other computing devices in industry, trade, and commerce as well as in engineering; and if our schools are to continue to be useful in training the future men and women for these occupations, this fact should be kept in mind.

Possibly one reason that we may balk at the idea of teaching the slide rule at the junior high level may be a reflection of the tendency some of us seem to have to want to teach everything on the page just because it's printed there. We may assume that the logical steps in presenting anything are necessarily the psychological ones. This is wishful thinking. Of course, the junior high is no place to begin slide rule work if one assumes that I suggest that we teach all the scales and all the uses of the slide rule just because all the scales happen to be on the rule, or simply because we, as teachers, happen to know all the uses of the instrument.

Yet, I believe the slide rule can be effectively used at this level, not as an end, but as a means. The mere fact that the slide rule shows no decimal point makes it

mandatory for the student to judge which number the significant figures represent. For example, does the "365" which the rule indicates as the answer mean 365 or 3.65 or 3650 or 0.0365 or what? Here is not only practice in estimating in a real situation, but practice which *must* be indulged in to get the answer. The lazy student has no effective way of "goldbricking" out of judging. Either he estimates the answer or he gets no answer.

At the Carroll Manor School, we have been using largely the C and D scales for multiplication and division. We have also done some work on squares and cubes of numbers as it fits in with units on measuring areas and volumes.

Slide rule work presupposes a good understanding of decimals; so the slide rule poses a possible motivation for "brushing up on our decimals so we can see the point." Obviously, the slide rule is made to order for work in metric units and can therefore show quite clearly the advantages of the metric system over the English one. Also comparisons between slide rules and between the slide rule answers and the "longhand" answers provides a good opportunity to introduce or review the idea of tolerances.

Best of all, students seem to show lasting interest in their "slipsticks," possibly since they feel that "only the guys who know what the score is—like engineers—use slide rules!" Experiments with these mechanical aids to computation can be guided into interesting side-lights: it is only a step from the standard slide rule to lead the students to see that, by substituting two linear scales (metric scales are best, since the decimal idea can be carried out) in place of the logarithmic scales one can do addition and subtraction using the mechanical principle of the slide rule.

Those students of the present classes who will use the slide rule a few years from now in trigonometry will be familiar with its working. They will have a decided advantage over those who must learn trigonometry and the slide rule at the same time. A much more current advantage, however, is seen in the significant decline I have noticed in the number of careless errors on the mathematics papers I'm marking these days.

Illinois Conference

THE ANNUAL Conference on the Teaching of Mathematics at Illinois State Normal University will be held Saturday, April 23. This is scheduled from 9:00 A.M. to 3:00 P.M. and sponsored by the university department of mathematics.

There will be discussion groups centered around the problems of teaching mathematics in elementary and secondary schools. Particular attention will be given to participation in the Illinois secondary school curriculum program sponsored by the Office of the Superintendent of Public Instruction.

The speaker for the elementary session will be Dr. H. Van Engen, head of the department of mathematics, Iowa State Teachers College, Cedar Falls. Speaker for the secondary session will be Dr. Harold P. Fawcett, chairman of the department of education, Ohio State University, Columbus.

This is the second year such a conference has been arranged by ISNU teachers. Some 300 mathematics instructors from all parts of Illinois attended the conference held last spring.

EDITORIALS

A Note of Appreciation

No one could fail to appreciate such an honor as was bestowed upon the editor of The Mathematics Teacher by the Board of Directors of The National Council of Teachers of Mathematics by making a special number of the January issue. Undoubtedly Professor Clark, the editor for that issue, would have liked to ask many other important leaders in the field to contribute to that issue, but he was limited in space. As a matter of fact, he did ask some people to contribute articles who, for one reason or another, were not able to get the articles written on time, if at all. To the Board, to Professor Clark, and to all the contributors the editor wishes to take this opportunity to express his sincere appreciation of their thoughtfulness.

What the editor has done in his almost twenty-five years of continuous service to the Council in getting out The Teacher and the *Yearbooks* is no more than many other interested members of the Council would gladly have done in order to help improve mathematical education.

W.D.R.

High School Mathematics

It might help the present situation in Secondary Mathematics if we tried to give all pupils a common experience with what might be termed the absolutely essential mathematics that the well educated citizen should know and then take care of individual differences in needs, interests, and abilities in two ways. First, by providing enrichment courses for superior pupils or for those who at least plan to go to college or to take up scholarly pursuits. And, secondly, by setting up certain specialized courses for the non-college or non-academic pupils who plan to follow some special line of work.

In this way, we could avoid the terms "second track," "third track" and the like, which tend to stigmatize the course so labelled or, at least, to make the pupils and also their parents feel that the course is inferior to the "first track" course which is taken by the more gifted pupils. The idea is at least worth considering before practice becomes standardized on the first and second track levels. It also seems reasonable to expect that the enrichment courses and the specialized courses might more nearly meet the needs of all pupils concerned.—W.D.R.

Institute for Teachers of Mathematics

Sponsored By
THE ASSOCIATION OF TEACHERS OF MATHEMATICS IN NEW ENGLAND

To be held on the Wellesley College Campus, Wellesley, Massachusetts, August 23-30, 1949

This Institute is for teachers of Mathematics in elementary schools, secondary schools, and colleges. Its purpose is to keep teachers up-to-date in the mathematics they teach, to make it possible for them to share ideas concerning methods used in teaching, to acquaint them with mathematical activities near Boston, and to enable them to relax among congenial and friendly surroundings.

Recreational activities are being planned to make your visit to New England enjoyable as well as educational. In addition to those contacts which you will have as part of the organized work on the campus, there will be time set aside in the program for recreation and for group or private

(Continued next page)

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR

Midwood High School, Brooklyn, New York

The American Mathematical Monthly

June–July, 1948, Vol. 55, No. 6.

1. Oldenburger, Rufus, "Practical Computational Methods in the Solution of Equations," pp. 335–342.
2. Moulton, E. J., "Two Teasers for Your Friends," p. 342.
3. Redheffer, Raymond, "A Machine for Playing the Game of Nim," pp. 343–349.
4. Salzer, H. E., "Further Remarks on the Approximation of Numbers as Sums of reciprocals," pp. 350–356.
5. Mathematical Notes, pp. 357–360.
   a. Thébault, Victor, "On the Twelve Point Sphere of the Tetrahedron."
   b. Niven, Ivan, "Note on a Paper by L. S. Johnston."
   c. Klee, Jr., V. L., "A Generalization of Euler's ϕ-Function."
6. Classroom Notes, pp. 360–364.
   a. Wagner, R. W., "Equations and Loci in Polar Coördinates."
   b. Green, L. C., "Maximum Uncertainty as a Simple Example of a Non-Distributive Algebra."
7. Elementary Problems and Solutions, pp. 364–368.
8. Advanced Problems and Solutions, pp. 369–373.
9. Recent Publications, pp. 374–377.
10. Clubs and Allied Activities, pp. 378–381.
11. News and Notices, pp. 382–384.
12. Official Reports and Communications, pp. 385–392.

The Mathematical Gazette

July 1948, Vol. 32, No. 300.

1. Our Three-Hundredth Number, pp. 97–98.
2. Langley, E. M., "The Eccentric Circle of Boscovitch," pp. 99–109. (Reprint)
3. Whitehead, A. N., "The Aims of Education —A Plea for Reform," pp. 110–119. (Reprint)
4. Heath, Sir T. L., "Greek Mathematics and Science," pp. 120–133. (Reprint)
5. Hardy, G. E., "The Case Against the Mathematical Tripos," pp. 134–145. (Reprint)
6. Greenstreet, W. J., "Review of MacFarlane's 'Ten British Mathematicians,'" pp. 146–152. (Reprint)
7. Russell, Bertrand, "Review of Keynes' 'A Treatise on Probability,'" pp. 152–159. (Reprint).
8. Siddons, A. W., "Some Notes on the A.I.G.T. and the M.A.," pp. 160–162.
9. Littlewood, J. E., "Large Numbers," pp. 163–171.
10. Durell, C. V., "The Transition from School to University Mathematics," pp. 172–178.
11. Littlewood, J. E., "Newton and the Attraction of a Sphere," pp. 179–181.
12. Bell, E. T., "The Basic Lemma in Multiplicative Diophantine Analysis," pp. 182–183.
13. Neville, E. H., "The Tricircular Generation of a Focus-Sharing Triad of Conics," pp. 184–186.
14. Littlewood, J. E., "Experiment and the Teaching of Mechanics," (Correspondence) p. 186.
15. Knight, R. C., "The Elementary Mathematics of the Rocket," pp. 187–194.
16. Robson, A., "Conics Which Touch Five Given Points," pp. 195–197.
17. Goodstein, R. L., "Proof by Reductio ad Absurdum," pp. 198–204.
18. Mathematical Notes, pp. 205–212.
19. Book Reviews, pp. 213–224.

Mathematics Magazine (Formerly *National Mathematics Magazine*)

January–February 1948, Vol. 21, No. 3.

1. Murnaghan, F. D., "The Operational Calculus," pp. 117–138.
2. Huff, Gerald B., "A Novel Algorithm at the Freshman Level," pp. 138–144.
3. Reviews of Current Papers and Books, pp. 145–146.
4. Thébault, Victor, "A French Mathematician of the Sixteenth Century—Jacques Peletier (1517–1582)," pp. 147–150.
5. Clark, C. E., "Proofs of the Fundamental Theorems of Spherical Trigonometry," pp. 150–155.
6. Problems and Questions, pp. 156–160.
7. Mathematical Miscellany, pp. 160–163.

Scripta Mathematica

June 1948, Vol. 14, No. 2.

1. Court, Nathan Altshiller, "The Tetrahedron and Its Altitudes," pp. 85–97.
2. O'Brien, Katherine, "The Present," p. 98.
3. Gardner, Martin, "Mathematical Card Tricks," pp. 99–111.
4. Baravalle, H. v., "Transformation of Curves by Inversion," pp. 113–125.
5. Mirsky, L., "A Remark on D. H. Lehmer's Solution of the Tarry-Escott Problem," pp. 126–127.
6. Joffe, Judah A., "Old and New Proofs of the Pythagorean Theorem," (no. 11), p. 127.
7. Boyd, Rutherford, "Mathematical Ideas in Design," pp. 128–135.
8. Book Reviews, pp. 136–150.
9. Recreational Mathematics, pp. 151–166.
10. Miscellaneous Notes, pp. 167–172.

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trips. The proximity of beautiful beaches and scenic mountains make New England an ideal place to visit.

The speakers who are scheduled for the Institute and their respective topics are:

Dr. Shields Warren, Pathologist at New England Deaconess Hospital and Professor of Pathology at Harvard Medical School. "Mathematics in Medical and Biological Research."

(*Continued next page*)

Bruce H. Billings, Director of Research of Baird Associates. "Mathematics in Chemical Research."

Edwin H. Land, President, Polaroid Corporation of America. "Mathematics of Optical Research."

Joseph Hilsenrath, Scientific Educational Advisor of the National Bureau of Standards. "Uses of Mathematics in Government Work."

Domina Eberle Spencer, Assistant Professor of Physics at Brown University. "Dimensional Analysis."

Herman H. Goldstine of the Electronic Computer Project, Institute for Advanced Study at Princeton. "High Speed Calculators and their Effects on Mathematics."

A member of the General Electric Company's Research Laboratory. "Mathematics in Electrical Research."

Chalmers L. Weaver, Assistant Actuary, New England Mutual Life Insurance Company. "How Insurance Companies Depend Upon Mathematics."

Harold A. Freeman, Associate Professor of Statistics, Massachusetts Institute of Technology. "Mathematical Statistics."

Garrett Birkhoff, Professor of Mathematics, Harvard University. "Foundations of Geometry."

The speaking program will serve to highlight a variety of Discussion Groups wherein teachers of mathematics at any level may find material of interest and value. The Discussion Group Topics and their Leaders are as follows:

1. "Teaching 'Why' Before 'How' in Elementary School Mathematics." Vincent J. Glennin, Professor of Education, Syracuse University.
2. "Teaching Algebraic Techniques Through Application." W. W. Rankin, Professor of Mathematics, Duke University.
3. "The Common Interests of High School and College Mathematics." Ralph Beatley, Associate Professor of Education, Harvard Graduate School of Education.
4. "Materials for Junior High School Mathematics." Veryl Schult, Head of Department of Mathematics, Divisions 1-9, Washington, D. C. Public Schools.
5. "A Program in General Mathematics and Its Placement." William Betz, Specialist in Mathematics, Rochester, N. Y.
6. "Problems in Consumer Mathematics." Mary A. Potter, Supervisor of Mathematics, Rachine, Wisconsin.
7. "Meeting Objectives in Geometry for Academic and General Education." Rolland R. Smith, Coordinator of Mathematics, Public Schools, Springfield, Massachusetts.
8. "Using Activities in Grade XII Mathematics." Howard F. Fehr, Professor of Mathematics, Teachers College, Columbia University.
9. "Field Work in Mathematics." Carl N. Shuster, Professor of Mathematics, State Teachers College, Trenton, New Jersey.

A mathematics laboratory where a variety of aids to teaching will be on display models may be made, and films and filmstrips viewed will be an important feature of the Institute. Consultants for the Laboratory are:

Lieutenant Colonel Robert C. Yates, Mathematics Department, West Point.

Christina S. Little, Acting Head of Mathematics Department, Girls High School, Boston, Massachusetts.

John F. Schacht, Mathematics Department, Bexley High School, Columbus, Ohio.

Elwood M. Stoddard, Mathematics Department, Hingham High School, Hingham, Massachusetts.

The closing event of the Institute will be a banquet at which Julian L. Coolidge, Professor of Mathematics at Harvard University will be the featured speaker.

Although no college credit will be given for the Institute, a certificate of attendance will be awarded participants. Tuition is free. Registration Fee is $10.00 for full-time participants. Room and Board are available at reasonable prices.

If you are looking for a good place to go for some late-summer fun and recreation while enjoying the refreshing experience of associating with others make this Institute a must on your summer program. You can leave your notebooks and pencils at home too, because a full report of the Institute will be forthcoming shortly after its conclusion.

Full details on all matters pertaining to the Institute and an application for registration are included in the program which is now available. Send for your copy now. Write to: Henry W. Syer, 84 Exeter Street, Boston 16, Massachusetts.